

## Study Of Boundary Layer Flow With Natural Convection Boundary Condition

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### Abstract

*In this paper we numerically study the nonlinear boundary value problem. We convert the nonlinear partial differential equation into nonlinear ordinary differential equation by taking suitable similarity variables and the solve by rungekutta fourth order method with shooting method. In this paper we particularly investigated the electric and magnetic effect on these equations.*

**Keywords:** - Similarity variables, Shooting Method, Runge kutta.

### 1- INTRODUCTION

A type of flow above a stretching sheet is major difficulty in several engineering procedures i.e.s. melt spinning, extrusion, warm / hot rolling, glass-fiber construction, plastic production, rubber sheets and many other. Polynomial sheets and filaments are created by endless extrusion of polymer as/of windup roller from a die, that's arranged at a fixed distance away. Across a surrounding fluid the thin polymer sheet represents a continuously moving / active surface with a non-uniform velocity [1]. Experiment represent that the velocity of the stretching sheet is proportional to the distance from the orifice [2]. Crane [3] studied the steady 2-dimensional incompressible boundary layer flow of a Newtonian fluid because of the stretching of a flexible flat sheet which moves in its own plane due to the application of a uniform stress with a velocity linearly with distance from a fixed point. This issue/ difficulty is particularly interesting since crane get an exact solution of the 2-dimensional Navier-stokes equation. After this work, in this

problem the flow field above a stretching sheet takes influential attention and a significant amount of literature has been generated [4-9]. [10] The heat transfer capacity/ability of convection heat transfer fluid like water, oil is very weak/low causing the thermal conductivity of these fluids. The experiments show that when we include nanoparticles in these base liquids then we can improve the thermal conductivity of fluids because it plays an important role on the heat transfer rate. [11] Using nanoparticles in the base fluid is an innovative idea. Nanotechnology has been extensively used in industry due to the fact that substance with sizes of nanoscale particle particular chemical and physical residence. [12]Choi et al. express/signified that when we include some little quantity of nanoparticles in conventional heat liquid then this experiment increases thermal conductivity of the fluid approximate two times. Khanafer et. Al. [13] is the first person who tested heat transfer capability of nanofluids inside a bounded area considering solid particle disunion. After the

nanotechnology is being introduced by these author's it is considered as one of the important forces that take vital industrial revolution of this century. The goal of nanotechnology is to manipulate the structure of the matter at the molecular level for getting the aspiration of innovation in Biology, Physics, Transportation, national security, environment, electronics and many other fields [14]. [15] Inside a straight heated tube and a radial space between heated disks and coaxial, the hydrodynamic and thermal characteristics of the laminar forced nanofluid convection flow were investigated [15]. [16-17] Between Brownian motion and thermophoresis, which are two free nanoparticles surfaces that do not change the convective cell's geometric configuration [16-17]. [18-19] The effect of using different nanofluid on natural convection flow, the numerical study was performed. Higher Rayleigh numbers can be extended for various types of

## 2- NOMENCLATURE

$u, v$  velocity components along  $x, y$ - axis  
 $\alpha$  Thermal diffusivity of Nano fluid  
 $u_w$  linear velocity with stretching sheet  
 $Pr$  Prandtl number  
 $\rho$  fluid density  
 $\lambda$  slip parameter  
 $T_{fluid}$  temperature  
 $c_p$  specific heat at constant pressure  
 $(\rho c_p)$  heat capacity  
 $U$  velocity of fluid  
 $C_f$  local friction coefficient

## 3- FLOW FIELD ANALYSIS

In this study, an incompressible nanofluid passing through a slip boundary a slip boundary stretching sheet. On the basis of Ohm's law and Maxwell's equation with study of EMHD formed the continuity equation, momentum equation, fluid energy equation, fluid concentration equation and sheet conduction equation in three dimensions. We suppose the flow is in the  $X$ -direction and the  $x$ -direction is along the

nanofluid [18-19]. [20] A global overview of convective transport in nanofluids assembled by Buongiorno [20] and Kakac and Pramuanjaroenkij [10]. Kuznetsov and Nield [21] have used the model in which Brownian motion and thermophoresis are considered and thus from this they deliberated the effect of nanoparticles on natural convection boundary-layer flow past a vertical plate. [22] The simplest possible boundary conditions are assumed by authors, in which the temperature and the nanoparticles fraction both are constant along the wall. Afterwards, Nield and Kuznetsova examined the Cheng-Mankowitz [23] problem of natural convection past a vertical plate in a porous medium saturated by a nanofluid. This model is taken into account for nanofluid incorporates the effects of Brownian motion and thermophoresis. Darcy model is applied in case of porous medium [24].

$f'(\eta)$  dimensionless velocity  
 $T_w$  temperature at the stretching surface  
 $\theta(\eta)$  dimensionless temperature  
 $\tau_w$  shear stress at surface  
 $T_\infty$  free convection  
 $q_w$  heat flux at surface  
 $\nu$  kinematic viscosity of fluid  
 $Re$  local Reynolds  
 $E$  dimensionless electric parameter  
 $M$  dimensionless magnetic parameter

thin plate in the rising direction and the  $y$ -axis is normal to it. To represent the buoyancy term, we use the Boussinesq approximation and we also include the electrical conductivity effect, heat source and the magnetic force effect. The Nano energy conversion thermal system is also used in it. For stretching sheet, the fluid motion within the film is due to a slip boundary layer. To make it a type of Nano particle mixed with fluid thermal system we

study the Brownian motion and thermophoresis effects model. For this fluid, in the usual notation, the steady two-dimensional boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_\infty \frac{\partial u_\infty}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho} (U - u) + \frac{\sigma B_0 E_0}{\rho} \quad (2)$$

Above equations are leading for the electrical magneto fluid dynamics flow field, in which x & y represents the Cartesian coordinates alongside the sheet and regular to it. In the x and y directions the velocity components of the fluid are u and v. By the natural stream velocity, the fluid flow towards the perpendicular plat layer is define from the expression  $u_\infty \frac{\partial u_\infty}{\partial x}$ . The fluid flow and heat transfer effect are better than any other implicit flow field because of the above expression is much bigger and give us the hundred present fluid flow amounts to the plat sheet. The expression can be modified as  $u_\infty \frac{\partial u_\infty}{\partial x} \sin\theta$ , the term will be vanished when we take the value of  $\theta=0$ , and it becomes a flat plate flow, and when we take the value of  $\theta=90$  degree, then it a stagnation flow, the term is  $u_\infty \frac{\partial u_\infty}{\partial x}$ . With a changeable magnetic parameter  $B_0$ , the electrically conducting fluid past a flat heated sheet and the electrical conductivity is  $\sigma$  which is a assumed constant, the electrical field factor is  $E_0$ , the fluid temperature is denoted by T,  $\infty$  denotes the natural stream notation,  $\rho_r$  denotes the density of fluid and the fluid's kinematic viscosity is  $\nu_r$ .

The boundary conditions are

$$\text{at } y = 0 \quad u = u_w = cx + L \frac{\partial u}{\partial y}, \quad v = \nu_w = ax$$

$$\text{at } y \rightarrow \infty \quad u \rightarrow 0 \quad (3)$$

Where L denotes the slip, parameter associated to the surface stretching reference velocity, the wall notation is w, the

proportional constant are a and c. New similarly variables define as

$$\eta = y \sqrt{\frac{a}{\nu_f}}, \quad u = axf'(\eta) \quad \& \quad v = -\sqrt{a\nu_f}f(\eta) \quad (4)$$

The non-dimensional temperature is defining as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (5)$$

By substituting equation 1 and 2 we get

$$f''' + ff'' - (f')^2 + M(1 - f') + ME = 0 \quad (6)$$

With boundary conditions (3) becomes

$$f(0) = 0, \quad f'(0) = 1, \quad \theta = 1, \quad \text{at } \eta = 0$$

$$f'(\eta) \rightarrow 0, \quad \theta \rightarrow 0, \quad \text{at } \eta \rightarrow \infty \quad (7)$$

Where dimensionless magnetic parameter signifies

by  $M = \frac{\sigma B_0^2}{\rho_f a}$ , and the dimensionless electric parameter signifies by  $E = \frac{E_0}{B_0 U}$ .

#### 4- HEAT CONVECTION ANALYSIS

For temperature T, the equation of the energy by regular usual boundary layer estimate on the occurrence of heat source, is given by

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{(\rho c_p)} (T - T_\infty) \quad (8)$$

Where the density is  $\rho$ , at constant pressure the specific heat of the fluid is denotes  $c_p$ , the heat capacity of fluid denotes by  $\rho c_p$ , the thermal diffusivity defines by  $\alpha_m = \frac{k}{\rho c_p}$ , the dimensionless heat generation coefficient is  $Q_0$ . By choosing appropriate boundary conditions we can find similarity solution of above equation

$$\frac{1}{Pr} \theta'' + f\theta' + \lambda\theta = 0 \quad (9)$$

Where Pr is the Prandtl number which is  $Pr = \frac{\nu}{\alpha}$ , and the dimensionless heat source or sink parameter is  $\lambda = \frac{Q_0}{a(\rho c_p)}$ . At present, the

physical significance of the term is composed of many factors, so that the motion of the solid particles at the fluid phenomena is expressed. Many compose

factor values are made from the Brownian motion coefficient. Includes the governing equations of this study so that the transfer of heat mass and fluid flow can be affected. On the other hand, how was the Brownian motion coefficient assessed? It depends on the coefficient of the Brownian motion to evaluate its compound factors.

### 5- NUMERICAL METHOD

In the present thermal energy application problem, an energy conversion system for heat conduction, convection system has been investigated. Since equations (6) & (9) are extremely non-linear system and finding a closed form or exact solution is difficult.

An approximate solution method has been used in this study to solve the problem. An improved method for solving the set of similarity equations was developed in present study was used an approximate solution method to solve the problem. We use Runge-Kutta method, similarity transformation method and shooting method for solving non-linear differential equation numerically.

### 6- RESULT AND DISCUSSIONS

In this study, the model for nanofluids passing a slip boundary stretching sheet Nano energy conversion thermal system was presented. Present Nano energy conversion effects non-dimensional parameters including Magnetic parameter (M), Electric parameter (E), Prandtl number (Pr), heat sink parameter ( $\lambda$ ) are mainly interested of the study. The stretching sheet flow and temperature fields were analyzed using the boundary layer concept to obtain a set of connector fluid momentum equation, energy equation. On the other hand, a heat conduction equation was used to solve the problem for the sheet plate heat conduction part. In present study; we use similarity

transformation to solved equation of continuity and equation of momentum which is equation (1) and (2) in the form of equation (6) with the help of boundary conditions (3) and similarity variables (4). Fig-1. Same system we use for equation (8) to find equation (9). Now we use Runge-Kutta method, similarity transformation method and shooting method for solving non-linear differential equation numerically. Then we find some graph.

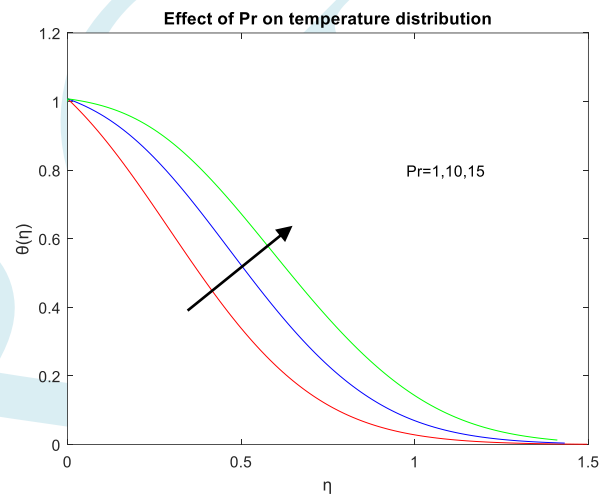


Fig-1

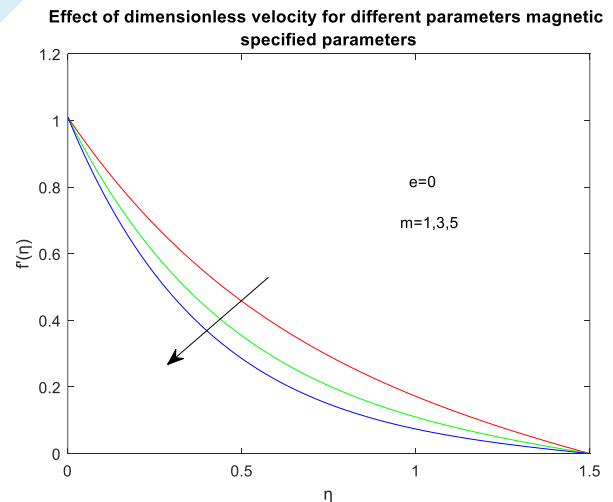
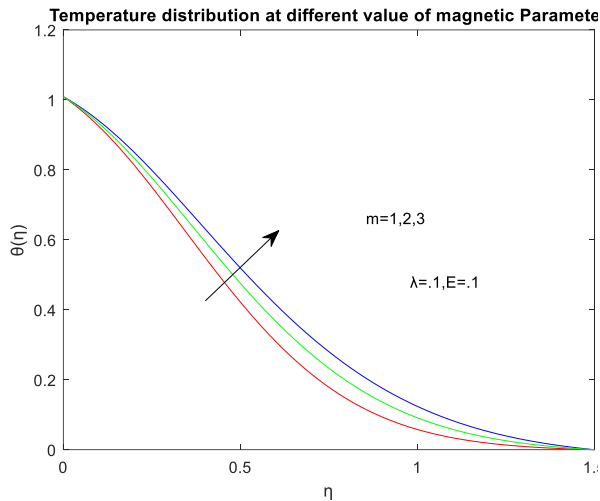
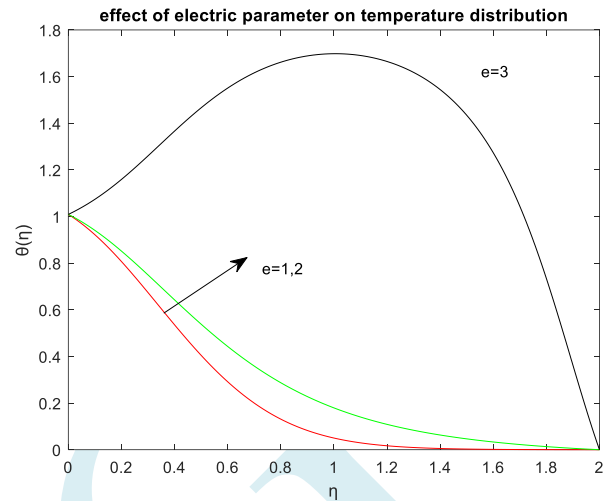


Fig-2



**Fig-3**



**Fig-4**

## 7- CONCLUSION

In this paper the problem defined above is solved numerically by using Runge Kutta with shooting method. Particularly we see the effect of magnetic and electric parameter on velocity and temperature. This study is depending on a novel Nano-energy conversion problem about extrusion stretching sheet heat transfer. On the other

1. In figure-1 it seems that when we increase Prandtl number, temperature is also increasing.
2. In figure-2 as magnetic parameter increase velocity is decreases.
3. In figure-3 at  $\lambda=1$  and  $E=1$  when we increase magnetic parameter

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hand, in order to express the function of the entire system, all the physical meaning of different status types, the paper uses the multimedia methods to investigate the related problems and at the same time obtain the results of numerical precision calculation. The conclusion of the problems is as follows:

4. In figure-4 when we increase the value of electric parameter respectively 1 and 2 temperature increase but in case of  $e=3$  temperature drastically increases.
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