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A Review On Reconstruction- Clustering –Of Fuzzy Images

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Abstract
In this paper fuzzy relation equations are used for compression/decompression process of color images. The results of the reconstructed images are compared. The compression is performed using fuzzy relation equations of min-t type, where t stands for the t-norm. The decompression process is realized via fuzzy relation equation of max-t type. And root means square error (RMSE) and peak signal noise ratio (PSNR) have been evaluated and compared the decompressed images with respect to the original image under different compression rates. Also the properties of images as clustering in images are studied under the Pattern recognition.
***Keywords: -** Fuzzy theory, Clustering, Matlab.*

1- INTRODUCTION

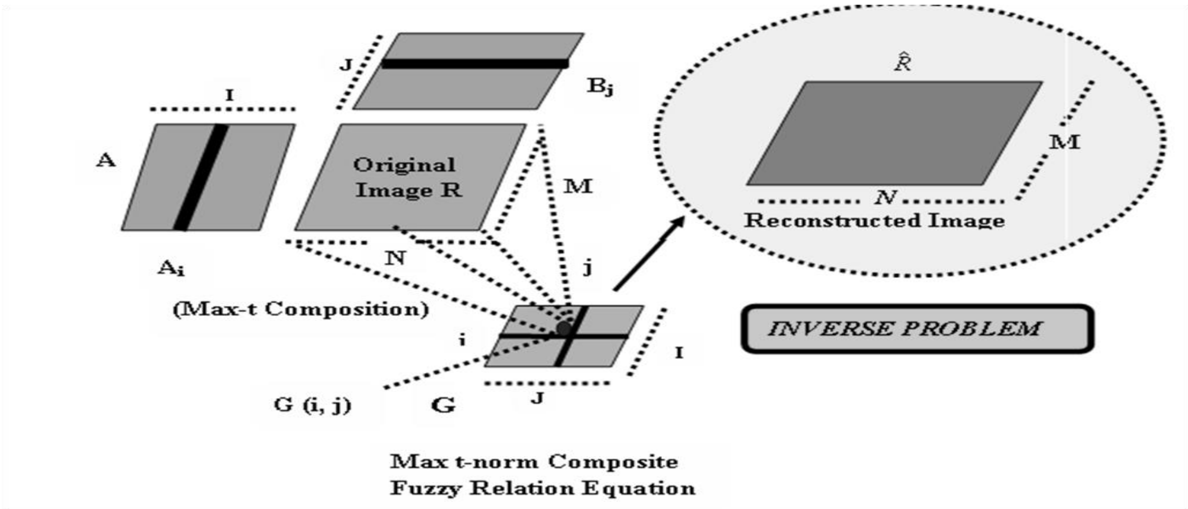
In science, we are dealing with uncertainty and which should be avoided by all possible means. In 1965 Zadeh [1] has given the concept of fuzzy set theory which is very much useful to deal with the uncertainty.

Image Processing is a method to convert an image into digital form and perform some operations on it, in order to get a strengthen image or to extract some useful information from it. It is a type of signal dispensation in which input is image, like video frame or photograph and output can be image or characteristics associated with that image. By this method we can observe the objects that are not visible in image, create a better image and distinguish the objects in an image.

Fuzzy image processing is the collection of all approaches that understand, represent and process the images, their segments and features as fuzzy sets. The representation and processing depend on the selected fuzzy technique and on the problem to be solved. A Pictorial object is a fuzzy set which is specified by same membership function defined on all picture points. Each image point participates in many memberships. Some of this uncertainty is due to degradation, some of its inherent. In fuzzy set terminology, making figure/group distinction is equivalent to transforming from membership functions to characteristics functions. Edge detection, clustering and segmentation etc are the properties of an image.

Here we have used the fuzzy relation equation for the compression and decompression of fuzzy images.

2- IMAGE COMPRESSION AND DECOMPRESSION PROCESS



A method of lossy image compression and reconstruction in setting of fuzzy relational equation is proposed ,where a still gray scale image is expressed as a fuzzy relation $R \in F(X \times Y)$, $X = \{x_1, x_2, \ldots, x_M\}$, $Y = \{y_1, y_2, \ldots, y_N\}$ by normalizing the intensity range of each pixel into [0, 1], the image R is compressed into $G \in F(I \times J)$ by

$$G(i, j) = (R^T \Delta A_i)^T \Delta Bj$$

Where $i \in I = \{1,2,\ldots, I\}$, $j \in J = \{1,2,\ldots, J\}$, $I < M, J < N, Ai(\subset F(y))$, and Δ : max continuous t-norm composition. Obtaining a reconstruction image $\hat{R}$ is regarded to be an inverse problem, under the condition that the compressed image G and the families of fuzzy sets A and **B** are given [2].

2.1- IMAGE COMPRESSION PROCESS

Let $X = \{x_m : m = 1,2,\ldots, M\}$ and $Y = \{y_n : n = 1,2,\ldots, N\}$ be finite sets, $F(X) = \{A\}$ and $F(Y) = \{B\}$ be families of all fuzzy sets on X and Y, respectively, and R, an element of $F(X \times Y) = \{R : X \times Y \to [0,1]\}$, be a binary fuzzy relation between the sets X and Y .Here T denotes the transposition of a fuzzy se and fuzzy relation. Let gray-scale image of size $M \times N$ pixels be a fuzzy relation $R \in F(X \times Y)$ by normalizing the intensity range of each pixels into [0, 1].

Some important definitions are given below:

1. Continuous t-Norm: Let t-norm be a two place function

$$t : [0,1] \times [0,1] \to [0,1]$$

satisfying a collection of properties

(T-1) $0tx = 0, 1tx = x$

(T-2) $xty \leq ztw$ if $x \leq z$ and $y \leq w$

(T-3) $xty = ytx$

(T-4) $(xty)tz = xt(ytz), \quad \forall x, y, z, w \in [0,1].$

2. Max-Continuous t-Norm composition: Let $Q \in F(X)$ and $R \in F(X \times Y)$ .we define the max-continuous t-norm composition of R and Q as

$$T = R^T \Delta Q \ (\in F(Y))$$

where

$$T(y) = \max_{x \in X} \{R(x, y) t Q(x)\}.$$

3. Image compression: Let $A=\{A_i \in F(X): i \in I=\{1,2,\ldots,I\}\}$, $B=\{B_j \in F(Y): i \in J=\{1,2,\ldots,J\}\}$ and $R \in F(X \times Y)$. We Define the max continuous t-norm composition of R, A_i, and B_j as

$$G(i,j)=(R^T \Delta A_i)^T \Delta B_j$$
$$=\max_{y\in Y}\{\{\max_{x\in X}(R(x,y)tA_i(x))\}tB_j(y)\}\ (\in[0,1]$$

An image compression process is defined by (9) i.e. an image R of the size $M \times N$ pixels is compressed into image $G \in F(I \times J)$ of the size $I \times J$ pixels by the max-continuous t-norm compositions of R, A and B under the condition that $I < M$ and $J < N$

2.2- RECONSTRUCTION PROCESS (GREATEST SOLUTION)

The reconstruction of an image $\hat{R}$ from the compressed image G is considered to be an inverse problem, under the condition that A, B and G are given [4].

1. t-Relative Pseudo complement: We define the t-relative pseudo complement of a in b as

$$a\varphi_t b=\sup\{c \in [0,1]: atc \le b\}.$$

note that t-relative pseudo complement has the following properties:

(R-1) $a\varphi_t \max\{b,c\} \ge \max\{a\varphi_t b, a\varphi_t c\}$

(R-2) $at(a\varphi_t b) \le b$

(R-3) $a\varphi_t(atb) \ge b$

(R-4) $a \le b \Rightarrow c\varphi_t a \le c\varphi_t b$

(R-5) $a \le b \Rightarrow a\varphi_t c \ge b\varphi_t c$

for all $a,b,c \in [0,1]$.

2. t-Cartesian Product: Let $Ai \in F(X)$ and $B_j \in F(Y)$. we define the t-Cartesian product of A_i and B_j as

$$A_i \times_t B_j\ (\in F(X \times Y))$$

where

$$(A_i \times_t B_j)(x,y)=A_i(x)tB_j(y).$$

3. $\nabla-composition$: Let $A_i \times_t B_j \in F(X \times Y)$ and $c \in [0,1]$. We define the $\nabla - composition A_i \times_t B_j and c$ as

$$(A_i \times_t B_j)\nabla c\ (\in F(X \times Y)$$

Where

$$((A_i \times_t B_j)\nabla c)(x,y)=(A_i \times_t B_j)(x,y)\varphi_t c.$$

Proposition1: For every

$$R, R_1, R_2 \in F(X \times Y), Q \in F(X), A_i \in A(\subset F(Y)),$$
$$A_i \times_t B_j \in F(X \times Y), (i,j) \in I \times J, and G(i,j) \in [0,1],$$

we have (P-1) $R \le \bigcap_{(i,j)\in I\times j}(A_i \times_t B_j)\nabla\{(R^T \Delta A_i)^T \Delta B_j\}$

(P-2) $\{\{\bigcap_{(i.j)\in I\times J}(A_i \times_t B_j)\nabla G(i,j)\}^T \Delta A_i\}^T \Delta B_j \le G(i,j), \quad \forall (i,j) \in I \times J.$

(P-3) $R_1 \le R_2 \Rightarrow R_1^T \Delta Q \le R_2^T \Delta Q$

Theorem1: Let $A_i \in A(\subset F(X)), B_j \in B(\subset F(Y)), G(i,j) \in [0,1], R$ be the set of $R \in F(X \times Y)$ such that $(R^T \Delta A_i)^T \Delta B_j = G(i,j), (i,j) \in I \times J$, and

$$\hat{R} = \bigcap_{(i,j)\in I\times J} (A_i \times_t B_j) \nabla G(i,j)$$

where

$$\hat{R}(x,y) = \min_{(i,j)\in I\times J} \{(A_i(x) t B_j(y)) \varphi_t G(i,j)\}.$$

Then $\hat{R}$ is the greatest element of R.

Families of fuzzy sets A and B are defined by

$$A = \{A_1, A_2, \ldots, A_I\}$$

$$A_i(x_m) = \exp(-sh(\{(iM \div I) - m\}^2) \qquad m = (1,2,\ldots,M)$$

and

$$B = \{B_1, B_2, \ldots, B_J\}$$

$$B_j(y_n) = \exp(-sh(\{(jN \div J) - n\}^2) \qquad (n = 1,2,\ldots,N)$$

where the preferable range of the parameter sh(>0) is 0.01-0.05
Under the compression rate $= IJ \div MN$, each minimal solution has no more than $I \times J$ nonzero intensity pixels on the image of size $M \times N$ pixels [4].

2.3- ROOT MEAN SQUARE ERROR: $\sqrt{\frac{(R(x,y) - RM(x,y))^2}{M \times N}}$

2.4- PEAK SIGNAL NOISE RATIO: $20\log_{10} 255 / RMSE$

Value of each pixel of the original image is normalized, where $R(i,j)/255$ is the normalized value of each pixel.

Yagar t-norm: It is defined as $t(a,b) = 1 - \min(1, [(1-a)^{\omega} + (1-b)^{\omega}]^{1/\omega})$

$$a\phi_t b = \begin{cases} 1, a = b = 0 \\ b^a otherwise \end{cases}$$

3- RESULTS

Root mean square error and peak signal noise ratio computed for different images in different sizes by using Yagar t-norm

The results are as follows:

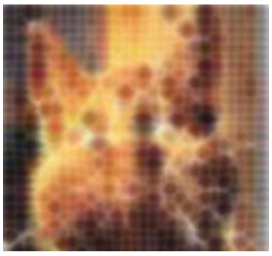
IMAGE NAME	ORIGINAL SIZE	COMPRESSED IMAGE	RMSE	PSNR
Cat	133	50	37.8165	16.577
Cat	133	30	30.753	18.370
Cat	133	90	24.999	11.442

Results of CAT image

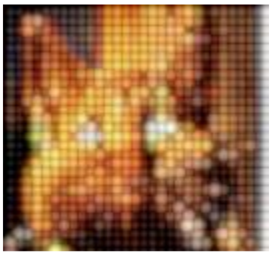
Original image 133x133pihels

Compressed image 30x30

.



Reconstructed image-maximum solution(133x133)



Reconstructed image–minimum solution (133x133)

Difference image (133x133)

Reconstructed image after adding difference image

3.1- CLUSTER

It is the assignment of objects into groups (called cluster) so that objects from the same cluster are similar to each other than objects from different clusters. Often similarity is assessed according to a distance measure.

Fuzzy Clustering: Given a finite set of data X, the problem of clustering in X is to find several cluster centers that can properly characterize relevant classes of X. In classical clustering analysis, these classes are required to form a partition of X such that the degree of association is strong for data those are within blocks of the partition and weak for data those are in different blocks. This requirement is too strong in many practical applications, and it is thus desirable to replace it with a weaker requirement. Fuzzy pseudo partition often called fuzzy c-partition, where c designates the number of fuzzy classes in the partition.

There are two methods of fuzzy clustering. One of them is based on fuzzy c-partitions, is called a fuzzy c-mean clustering method. The other method, based on fuzzy equivalence relations, is called a fuzzy equivalence relation-based hierarchical clustering method.

Fuzzy c-Means Clustering Method [5]:

Let $X = \{x_1, x_2, ..., x_n\}$ be a set of given data. A fuzzy pseudo partition of fuzzy c-partition of X, denoted by $P = \{A_1, A_2, ...A_c\}$, which satisfies

$$\sum_{i=1}^{c} A_i(x_k) = 1$$

For all $k \in N_n$ and

$$0 < \sum_{k=1}^{n} A_i(x_k) < n$$

For all $i \in N_n$, where c is a positive integer.

Given a set of data $X = \{x_1, x_2, ..., x_n\}$, where x_k is a vector

$$x_k = \{x_{k1}, x_{k2}, ..., x_{kp}\} \in R^P$$

For all $k \in N_n$, the problem of fuzzy clustering is to find fuzzy pseudo partition and the associated cluster centers by which the structure of the data is represented as best as possible. This enquires some criterion expressing the general idea that associations be strong within clusters and weak between clusters. To solve the problem of fuzzy clustering, we need to formulate this criterion in terms of a performance index. The performance index is based upon cluster centers.

Given a pseudo partition $P = \{A_1, A_2, ...A_c\}$, the c cluster centers, $v_1, v_2, ..., v_c$ associated with the partition are calculated by the formula

$$v_i = \frac{\sum_{k=1}^{n} [A_i(x_k)]^m X_k}{\sum_{k=1}^{n} [A_i(x_k)]^m}$$

For all $i \in N_c$, where m>1 is a real number that govern the influence of membership grades. The performance index of a fuzzy pseudo partition $P, J_m(P)$ is then defined in terms of the cluster centers by the formula

$$J_m(P) = \sum_{k=1}^{n} \sum_{i=1}^{c} [A_i(x_k)]^m \|x_k - v_i\|^2$$

Where $\|.\|$ is some inner product –induced norm in space R^P and $\|x_k - v_i\|^2$ represents the distance between $x_k and v_i$. This performance index measures the weighted sum of distance between cluster centers and element in the corresponding fuzzy clusters. Clearly, the smaller the value of $J_m(P)$, the better the fuzzy pseudo partition P. Therefore, the goal of the c-mean clustering method is to find a fuzzy pseudo partition P that minimizes the performance index $J_m(P)$. That is, the clustering problem is an optimization problem.

Results of fuzzy c-mean clustering

Image- 1

Reconstructed Image of Image-1 Using Lukasiewicz t-Norm

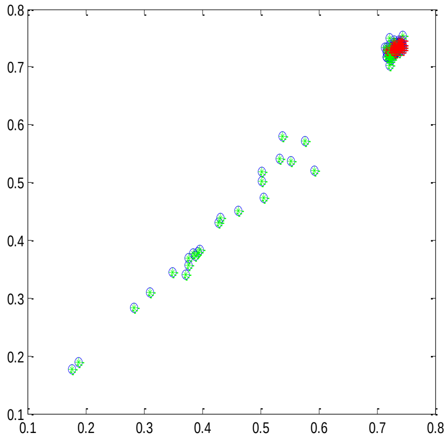


Image-2

Result obtain after applying fuzzy c-mean clustering to the Image-1

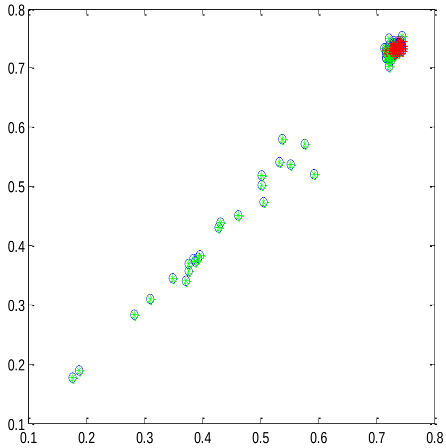
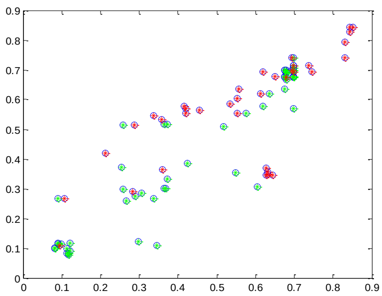
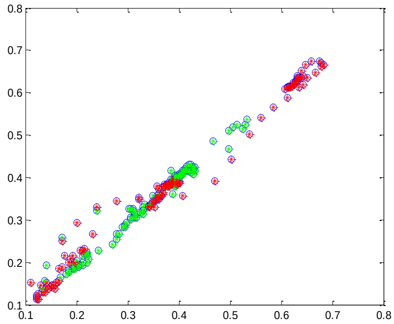


Image-3

The results of the fuzzy c mean clustering for the different pictures:





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