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Sensitivity Analysis Of Inventory Model For Deteriorating Items With On-Hand Inventory Dependent Demand Rate And Infinite Production Rate Without Shortages

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Abstract
A deterministic inventory model for deteriorating items is considered. The rate of demand is variable and it is on-hand inventory dependent. Rate of replenishment is instantaneous. Shortages are not allowed. The average total profit is maximized to decide the time of production run. Hence the economic lot size is obtained. The result is illustrated by various numerical examples with sensitivity analysis.
***Keywords: -** Sensitivity, inventory model, on-hand inventory, infinite production rate.*

1- INTRODUCTION

Inventory is a stock of items which is held to satisfy the future demands. Some items deteriorate with time. The process of deterioration is found in medicines, blood components, fashion items, vegetables, fruits and sweets etc.

Ghare and Schrader [1] solved the problem of decaying inventories exponentially and developed an EOQ model with constant demand. Covert and Philip [2] considered a two-parameter Weibull's distribution for variable rate of deterioration. Shah and Jaiswal [3] presented an order-level inventory model for a system with constant rate of deterioration. Aggarwal [4] calculated the average inventory holding cost. Park [5] gave an integrated production-inventory model for decaying raw materials. Hwang [6] optimized the production planning problem with continuously distributed time-lags. Yang and Wee [7] developed an integrated multi-lot-size production inventory model for deteriorating item. Sharma et al. [8] analyzed a deterministic production inventory model for deteriorating products with exponentially declining demand and shortages. Mishra and Shah [9] presented inventory management of time dependent deteriorating items with salvage value. Baten and Kamil [10] studied the inventory management systems for hazardous items of two-parameter exponential distribution with constant production and demand rates. Li and Mawhinney [11] presented a review on deteriorating inventory study. Manna and Chiang [12] solved economic production quantity models for deteriorating items with ramp type demand. Mishra et al. [13] developed a deterministic inventory model for deteriorating items with on-hand inventory dependent, variable type demand rate. Baten and Kamil [14] studied optimal fuzzy control with application to discounted cost production inventory planning problem. Mishra and Singh [15] gave computational approach to an inventory model with ramp-type demand. Mishra et al. [16] presented a partial backlogging EOQ model for queued customers with power demand and quadratic deterioration. Sharma and Muhammad [17] developed an EOQ Model for Hazardous Items of Two-Parameter Exponential Distribution with Uniform Rate of Demand & Finite Rate of Replenishment. Dash et al. [18] considered an inventory model for deteriorating items with exponential declining demand and time-varying holding cost. Sharma and Muhammad [19] analyzed an EOQ model with uniform demand & instantaneous production for hazardous items of two-parameter exponential distribution.

2- MODEL DESCRIPTION

The following notations and assumptions are used to derive this EOQ model:

2.1. Notations

I(t) = On hand inventory at any time t

d (t) = Demand rate of the item at any time t

D = Rate of change of demand rate with respect to t.

H = The constant deterioration rate where $0 \le H < 1$

I_0 = Inventory at time t = 0

C_p = Production cost of one item

P_s = Selling price per unit

C_h = Holding cost per unit per unit time

C_o = Operating cost per production run

T = Time of one production cycle

Q = Lot size per production run or cycle

d_0 = Constant demand rate up to a certain time t_1

$U(t - t_1)$ = Heaviside's unit step function.

I_1 = Inventory at time $t = t_1$

P = Average total profit per unit time

2.2. Assumptions

Deterioration of items is allowed with a constant rate (H).

The rate of demand (d) is variable with respect to time.

The rate of replenishment is infinite but size is finite.

Time horizon is finite.

There is no repair of deteriorated items will occur during the cycle.

Shortages are not allowed.

Lead time is zero.

The inventory system deals with only one item.

3- MATHEMATICAL MODEL

The behavior of the inventory system is describing by the differential equation

$$\frac{dI(t)}{dt} = -d(t) - H\cdot I(t) \qquad \dots (1)$$

$$\text{Where } d(t) = d_0 + D\cdot(t - t_1)\cdot U(t - t_1) \qquad \dots (2)$$

The solution of above system is obtained as

$$I(t) = -\frac{d_o}{H} + e^{-Ht}\left(I_o + \frac{d_o}{H}\right) \text{ for } 0 \le t \le t_1$$

$$\text{and } I(t) = \frac{D}{H}(T - t) \text{ for } t_1 \le t \le T \qquad \dots (3)$$

$$\text{with } I_1 = -\frac{d_o}{H} + e^{-Ht_1}\left(I_o + \frac{R_o}{H}\right) = \frac{D}{H}(T - t_1), \qquad \dots (4)$$

$$T = \frac{HI_1}{D} - \frac{1}{H}\log\left(\frac{d_0 + HI_1}{d_0 + HI_0}\right), \qquad \dots (5)$$

$$Q = \frac{1}{H}\left(I_o + \frac{d_o}{H}\right).(1 - e^{-Ht_1}) - \frac{d_o t_1}{H} + \frac{D}{2H}(T - t_1)^2 \qquad \dots (6)$$

$$\text{and } P = \frac{I_0}{T}(P_s - C_p - C_0) - \frac{C_h}{T}Q \qquad \dots (7)$$

The necessary condition for P to attain maximum is $\frac{dP}{dI_0} = 0$, which gives

$$I_0 = \frac{d_0(P_s - C_p - C_0)}{C_h - H(P_s - C_p - C_0)} \qquad \dots (8)$$

Using equation (8), $\frac{d^2P}{dI_0^{\,2}} = \frac{-C_h d_0}{T(d_0 + HI_0)} < 0$.

So it will give a global maximum for profit function P (I_0).

4- NUMERICAL EXAMPLES

4.1 Taking the values of parameters t_1 = 0.20 year, I_0 = 350 units, d_0 = 300 units/year, D = 60 units/year^2, P_S = $30/unit, C_p = $10/unit, C_h = $0.7 per unit per year, C_o = $12/set up, H = 0.2, the following results were obtained from our inventory model: T = 1.3098 year, I_1 = 277.5 units, Q = 216.666 units and maximum P = $2021.87 per year.

4.2 For different value of D and constant values of other parameters same as in example 4.1, the results are shown in the table 1.

Table 1. Effects of changes in D

D	I_1	T	Q	P
35	277.5	1.7855	282.653	1457.384
40	277.5	1.5873	255.158	1651.474
45	277.5	1.4332	233.774	1839.545
50	277.5	1.3098	216.666	2021.873
55	277.5	1.2089	202.669	2198.716
60	277.5	1.1249	191.005	2370.319

4.3 For different value of t_1 and constant values of other parameters same as in example 4.1, the results are shown in the table 2.

Table 2. Effects of changes in t_1

t_1	I_1	T	Q	P
0.1	313.4	1.3535	229.561	1950.03
0.15	295.3	1.3313	222.812	1986.057
0.2	277.5	1.3098	216.666	2021.873
0.25	259.8	1.2891	211.093	2057.435
0.3	242.3	1.2691	206.062	2092.7
0.35	224.9	1.2497	201.543	2127.622

4.4 For different value of I_0 and constant values of other parameters same as in example 4.1, the results are shown in the table 3.

Table 3. Effects of changes in I_0

I_0	I_1	T	Q	P
250	181.4	0.9255	108.891	2078.576
300	229.4	1.1177	158.163	2048.241
350	277.5	1.3098	216.666	2021.873
400	325.5	1.502	284.401	1997.949
450	373.5	1.6942	361.366	1975.639
500	421.6	1.8863	447.563	1954.448

4.5 For different value of d_0 and constant values of other parameters same as in example 4.1, the results are shown in the table 4.

Table 4. Effects of changes in d_0

d_0	I_1	T	Q	P
200	297.1	1.3883	241.167	1895.306
250	287.3	1.3491	228.725	1956.85
300	277.5	1.3098	216.666	2021.873
350	267.7	1.2706	204.992	2090.697
400	257.9	1.2314	193.703	2163.686
450	248.1	1.1922	182.797	2241.251

4.6 For different value of P_S and constant values of other parameters same as in example 4.1, the results are shown in the table 5.

Table 5. Effects of changes in P_S

P_s	I_1	T	Q	P
20	277.5	1.3098	216.666	-650.206
25	277.5	1.3098	216.666	685.8336
30	277.5	1.3098	216.666	2021.873
35	277.5	1.3098	216.666	3357.912
40	277.5	1.3098	216.666	4693.951
45	277.5	1.3098	216.666	6029.99

4.7 For different value of C_P and constant values of other parameters same as in example 4.1, the results are shown in the table 6.

Table 6. Effects of changes in C_P

C_p	I_1	T	Q	P
6	277.5	1.3098	216.666	3090.704
8	277.5	1.3098	216.666	2556.288
10	277.5	1.3098	216.666	2021.873
12	277.5	1.3098	216.666	1487.457
14	277.5	1.3098	216.666	953.0415
16	277.5	1.3098	216.666	418.6258

4.8 For different value of C_h and constant values of other parameters same as in example 4.1, the results are shown in the table 7.

Table 7. Effects of changes in C_h

C_h	I_1	T	Q	P
0.5	277.5	1.3098	216.666	2054.956
0.6	277.5	1.3098	216.666	2038.414
0.7	277.5	1.3098	216.666	2021.873
0.8	277.5	1.3098	216.666	2005.331
0.9	277.5	1.3098	216.666	1988.79
1	277.5	1.3098	216.666	1972.249

4.9 For different value of C_o and constant values of other parameters same as in example 4.1, the results are shown in the table 8.

Table 8. Effects of changes in C_o

C_o	I_1	T	Q	P
10	277.5	1.3098	216.666	2556.288
11	277.5	1.3098	216.666	2289.081
12	277.5	1.3098	216.666	2021.873
13	277.5	1.3098	216.666	1754.665
14	277.5	1.3098	216.666	1487.457
15	277.5	1.3098	216.666	1220.249

4.10 For different value of H and constant values of other parameters same as in example 4.1, the results are shown in the table 9.

Table 9. Effects of changes in H

H	I_1	T	Q	P
0.1	283.7	0.7673	143.811	3517.82
0.15	280.5	1.0416	181.08	2566.378
0.2	277.5	1.3098	216.666	2021.873
0.25	274.4	1.572	250.624	1669.54
0.3	271.4	1.8283	283.004	1423.129
0.35	268.4	2.0787	313.859	1241.287

5- CONCLUSION

In this paper sensitivity is analyzed for deteriorating items with on-hand inventory dependent demand rate and infinite rate of replenishment. The shortages are not allowed. The average total profit per unit time is maximized in various circumstances to decide the production cycle time and hence the optimum inventory level. The Karl-Pearson's coefficient of correlation between all the parameters and the variables is given in the last columns of the following tables 10, 11, 12 and 13.

Table 10. Coefficient of correlation between parameters and inventory at time $t = t_1$

D	t_1	I_0	d_0	P_s	C_p	C_h	C_o	H	I_1
35(5)60	0.2	350	300	30	10	0.7	12	0.2	-0.638876565
50	.1(.05).35	350	300	30	10	0.7	12	0.2	-0.999973335
50	0.2	250(50)500	300	30	10	0.7	12	0.2	1
50	0.2	350	200(50)450	30	10	0.7	12	0.2	-1
50	0.2	350	300	20(5)45	10	0.7	12	0.2	-0.638876565
50	0.2	350	300	30	6(2)16	0.7	12	0.2	-0.638876565
50	0.2	350	300	30	10	.5(.1)1	12	0.2	-0.638876565
50	0.2	350	300	30	10	0.7	10(1)15	0.2	-0.638876565
50	0.2	350	300	30	10	0.7	12	.1(.05).35	-0.999971601

Table 11. Coefficient of correlation between parameters and cycle time

D	t_1	I_0	d_0	P_s	C_p	C_h	C_o	H	T
35(5)60	0.2	350	300	30	10	0.7	12	0.2	-0.987690259
50	.1(.05).35	350	300	30	10	0.7	12	0.2	-0.999690133
50	0.2	250(50)500	300	30	10	0.7	12	0.2	1
50	0.2	350	200(50)450	30	10	0.7	12	0.2	-1
50	0.2	350	300	20(5)45	10	0.7	12	0.2	0.319438282
50	0.2	350	300	30	6(2)16	0.7	12	0.2	0.319438282
50	0.2	350	300	30	10	.5(.1)1	12	0.2	0.319438282
50	0.2	350	300	30	10	0.7	10(1)15	0.2	0.319438282
50	0.2	350	300	30	10	0.7	12	.1(.05).35	0.999861908

Table 12. Coefficient of correlation between parameters and EOQ

D	t_1	I_0	d_0	P_s	C_p	C_h	C_o	H	Q
35(5)60	0.2	350	300	30	10	0.7	12	0.2	-0.987690259
50	.1(.05).35	350	300	30	10	0.7	12	0.2	-0.997365063
50	0.2	250(50)500	300	30	10	0.7	12	0.2	0.995083562
50	0.2	350	200(50)450	30	10	0.7	12	0.2	-0.999711043
50	0.2	350	300	20(5)45	10	0.7	12	0.2	-0.654398096
50	0.2	350	300	30	6(2)16	0.7	12	0.2	-0.654398096
50	0.2	350	300	30	10	.5(.1)1	12	0.2	-0.654398096
50	0.2	350	300	30	10	0.7	10(1)15	0.2	-0.654398096
50	0.2	350	300	30	10	0.7	12	.1(.05).35	0.999407344

Table 13. Coefficient of correlation between parameters and maximum profit

D	t_1	I_0	d_0	P_s	C_p	C_h	C_o	H	P
35(5)60	0.2	350	300	30	10	0.7	12	0.2	0.999747257
50	.1(.05).35	350	300	30	10	0.7	12	0.2	0.999983838
50	0.2	250(50)500	300	30	10	0.7	12	0.2	-0.997781977
50	0.2	350	200(50)450	30	10	0.7	12	0.2	0.999107139
50	0.2	350	300	20(5)45	10	0.7	12	0.2	1
50	0.2	350	300	30	6(2)16	0.7	12	0.2	-1
50	0.2	350	300	30	10	.5(.1)1	12	0.2	-1
50	0.2	350	300	30	10	0.7	10(1)15	0.2	-1
50	0.2	350	300	30	10	0.7	12	.1(.05).35	-0.953912348

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