

### Visit Of Space: Quantum To Material Objects

**Dr. Shobha Lal**

Prof. of Mathematics and Computing, Jayoti Vidyapeeth Women's University, Jaipur,  
Rajasthan, India

Email- [dean.fet@jvwu.ac.in](mailto:dean.fet@jvwu.ac.in)

#### **Abstract**

*This research work is based on the concept of greatest American astrophysicist John oppenheimer who had proclaimed that in coming five Billion years there will be no Sun in the space, and the survival of all living bodies and other natural resources will be on stake. As per concept of Albert Einstein, velocity of the light is maximum and no other particle can move faster than velocity of light. In case of black hole, it has been an established fact that even bundles of light each compel to pass throw it. The total radiator light is absorbed by that very black hole. Kerr has been such a mathematician who has done a lot regarding energy extraction and radiation of energy from a black hole. According to this scientist if angular momentum  $a=0$  i.e. angular momentum is 0. This black hole is said to be a Kerr black hole. In the case of Kerr black hole the magnitude of spin quantum in supposed to be minimum and in between event horizons and ergo sphere. It has been an evident fact that fission and fusion process hues amount of energy is radiated from Kerr black hole*

**Keywords:** Space, Time, Kerr metric, Kerr black hole, Relativity.

#### **1- INTRODUCTION**

In this work different phases in the life of a star would be taken into the consideration. As in Indian Mathematical Works and societies as well as scientific societies there has been lesser work in the field of Astrophysics, Astronomy and cosmology, which have been traditionally backbone of the space science and space engineering. **Dr. S.Chandrashekharan**, the first Indian from Chennai whom got Nobel Prize in 1984 by finding out Chandrashekharan mass of a collapsing star that is a black hole. After his this prestigious achievement in spite of University of Cambridge, where there is a separate department of cosmology, Indian mathematicians also came forward to proceed further in the study and research of different celestial bodies. Present work is intended to give extension of the work of the Author who has been a Professor of mathematics and computing and the dean of faculty of engineering and technology

Jayoti vidhyapeeth women's university jaipur, whose topic of research has been " **A STUDY OF CIRCULAR ORBITS OF A TEST PARTICLE IN THE EQUATORIAL PLANE OF BLACK HOLE.**"(2002)-BRABU

**MUZAFFARPUR.** Under kind support of INDIAN SPACE RESEARCH ORGANISATION (ISRO)

BANGLORE.As per a recognized concept of John open heimer there will be no sun in the space, in coming five billion years then energy storage will be a vital challenge before the world and research scholars working for the development on the issue and saving of nature as a balanced ecosystem would be with empty hands. Due to gravitational pulling, process of fission and fusion collapsing of a star is a natural phenomenon and emission of heat and energy is an uncovered fact for everyone. How to store energy and utilized this storage energy for further concern is the point compelled us

to think over it and workout the solution. A research paper under the title “**RADIATION FROM PARTICLE FALLING INTO KERR BLACK HOLE**”, written by me and published in an International Journal (International Journal of Mathematical science and applications, volume no. 2 number 1, January 2012, page no. 481-490) has become a very clear cut and worldwide considerable fact that suggests further extension of the energy storage would taken, as suggested by me. In during the research work with my Research Scholar attempt by scholar’s side has been to publish some corollary parts of the research and procede further to get the approximated result by utilizing the proposed methodology.

## **2- OBJECTIVES**

The proposed research work is based on the concept of greatest American astrophysicist John oppenheimer who had proclaimed that in coming five Billion years there will be no Sun in the space, and the survival of all living bodies and other natural resources will be on stake. As per concept of Albert Einstein, velocity of the light is maximum and no other particle can move faster than velocity of light. In case of black hole, it has been an established fact that even bundles of light each compel to pass throw it. The total radiator light is absorbed by that very black hole. Kerr has been such a mathematician who has done a lot regarding energy extraction and radiation of energy from a black hole. According to this scientist if angular momentum  $a=0$  i.e. angular momentum is 0. This black hole is said to be a Kerr black hole. In the case of Kerr black hole the magnitude of spin quantum in supposed to be minimum and in between event horizons and ergo sphere. It has been an evident fact that fission and fusion process hues amount of energy is radiated from Kerr black hole.

The plan of process research is to suggest is to suggest such a safe zone in between event horizon and ergo sphere, where a

Skylab may be established to trape radiated energy from a Kerr black hole. The proposed research work bears a fast dimension as an objective. Tending towards the conservation of energy as per principle of energy conservation system and a solution to the world scientific organization that how, in absence of the Sun survival of universe is possible.

## **3- METHODOLOGY**

1. We study the formation and destruction of a star using Mathematical method applying tensor theory.
2. We generalized the general theory of relativity through functional analysis process.
3. We may use some Astronomical instruments outside the University campus (Astronomical Institute, Pune)

## **Information about work taken and performed on the Space**

### **A Brief History of Space Science In Japan**

Aryabhata was the first of the major mathematician-astronomers from the classical age of Indian mathematics and Indian astronomy. He is the author of several treatises on mathematics and astronomy, some of which are lost. His main works are Aryabhatiya and Arya-siddhanta. Aryabhatiya was particularly popular in South India, where numerous mathematicians over the ensuing millennium wrote commentaries. The work was written in verse couplets and deals with mathematics and astronomy. Arya-siddhanta circulated mainly in the northwest of India and, through Iran, had a profound influence on the development of Islamic astronomy. It is one of the earliest astronomical works to assign the start of each day to midnight. His contribution to mathematics: Place value system and zero: The place-value system, first seen in the 3rd-century Bakhshali Manuscript, was clearly in place in his work. While he did not use a symbol for zero, the French mathematician Georges Ifrah argues that knowledge of zero was implicit in Aryabhata's place-

value system as a place holder for the powers of ten with null coefficients. Value of pi: He also worked on the approximation for pi, and may have come to the conclusion that pi is irrational. In the second part of the Aryabhatiyam (gaṇitapāda 10), he writes:

caturadhikam śatamaṣṭaguṇam  
dvāṣaṣṭistathā sahasrāṇām  
ayutadvayaviṣkambhasyāsanno  
vṛttapariṇāhaḥ.

"Add four to 100, multiply by eight, and then add 62,000. By this rule the circumference of a circle with a diameter of 20,000 can be approached."

This implies that the ratio of the circumference to the diameter is  $((4 + 100) \times 8 + 62000)/20000 = 62832/20000 = 3.1416$ , which is accurate to five significant figures.

**Trigonometry:** He gave the area of a triangle as: tribhujasya phalashariram samadalakoti bhujardhasamvargah that translates to: "for a triangle, the result of a perpendicular with the half-side is the area."

He also discussed the concept of *sine* in his work by the name of *ardha-jya*, which literally means "half-chord".

#### **Algebra:**

In Aryabhatiya, he provided elegant results for the summation of series of squares and cubes:

It may be appropriate to explain how space science in Japan was initiated. Research involving all kinds of aerospace technologies had been banned after World War II until the ban was lifted in 1952 when Japan recovered its sovereignty. Prof. Hideo Itokawa of the Institute of Industrial Science, University of Tokyo, was inspired to develop rocket-propelled transport and initiated research into a solid-propellant rocket. He succeeded in developing the first tiny Pencil Rocket in 1955. In following years, his rocket was successively scaled up with the names Baby, Sigma, and Kappa.

The timing was good. The International Geophysical Year (IGY) organized by the

Committee on Space Research (COSPAR) in the period from 1957 to 1958 boosted the enthusiasm of Japanese space scientists. Owing to their strong efforts, they succeeded in participating in the IGY program with the Kappa rocket for aeronomy and ionospheric studies. Japan was the fourth country, following the United States, the Union of Soviet Socialist Republics, and the United Kingdom, to utilize sounding rockets for space research.

Such activities soon expanded beyond the scale that can be accommodated in one university laboratory. At the same time, the desire to have a central institution that served the entire Japanese space science community was growing strong. Eventually, this request obtained governmental support.

Unified development of space science in Japan started in 1964, when an institute for common use by Japan's universities (as explained below), the Institute of Space and Aeronautical Science (ISAS), was founded inside the University of Tokyo. Taking the preexisting Aeronautical Research Institute (since 1918) as the core, the groups of rocket technology and related space engineering/science were integrated in ISAS.

When the more powerful Lambda rocket became operational, ISAS was challenged to launch an artificial satellite. This venture encountered a number of difficulties. After painful days, they finally succeeded in placing a small satellite into orbit in 1970 (the same year as the Uhuru launch). Following Lambda, the Mu series of rockets were developed. Payload weight steadily increased. With the Mu rockets, ISAS firmly entered into the satellite era.

This era of ISAS, however, had fundamental problems. When I moved to ISAS in 1974, I observed that there was little scientific cooperation between the aeronautics people and the space engineering/science people. These two groups of people had different cultures and

interests. Also, as long as ISAS belonged to the University of Tokyo, there was little hope for further expansion. It became inevitable for space engineers and space physicists to seek independence from the University of Tokyo. This plea reached the Ministry of Education. Finally, the new national Institute of Space and Astronautically Science (ISAS), independent of the University of Tokyo, was established in April 1981. This gave Japanese space research a major boost.

Rocket development steadily continued at the new ISAS. The last model M-V has been operational since 1997. It had a low-Earth orbit capability of carrying a 1.8-ton payload. This may have been the world's largest all-solid propellant satellite launcher for the purpose of scientific research. The handling and launch operations of a solid-propellant rocket are much easier than they are for a liquid-fuel rocket. Solid propellant provides a significant advantage for launching scientific satellites ourselves.

It may be worth emphasizing that these rockets were the outcome of purely academic research. ISAS was a modest-scale institute, with a total staff numbering about 300 people. Yet, it was probably the only institute in the world that owned the technology of satellite launch vehicles and launch capability in full. The X-ray satellites (including solar X-ray satellites) launched by ISAS include Hakucho, Hinotori (by old ISAS), Tenma, Ginga, Yohkoh, ASCA and Suzaku (by the new ISAS).

As mentioned below, ISAS has become part of a bigger organization JAXA (Japan Aerospace EXploration Agency) since 2003. As one of the consequences of joining JAXA, M-V-rockets, the pride of ISAS, have become abolished. This is a great regret of many space scientists in Japan.

**The Space Science Policy of the Instituted Of Space and Astronautically Science**  
**Inter-University Autonomy**

ISAS is autonomously operated; all space engineers and space scientists of Japan elect members of the space engineering committee and the space science committee at ISAS. These committees deal with important issues of the space science program. Mission selection is one of them. Once a mission proposal is submitted, the committee evaluates its scientific significance and technical feasibility as well as the group's capability of conducting the mission concerned. A special group of experts is often assigned to further scrutinize the details.

There is an executive committee above the two space committees that serves as an advisory board to the Director-General on policy-related matters. In this way, all the space scientists of Japan commit themselves to the governance of ISAS. This system may be called open autonomy, and it is still maintained today even after the reorganization.

In 1975, the Ministry of Education consulted the Science Council (a group of "wise men") to develop a basic strategy for the ISAS space program. The Science Council submitted advice that could be called the constitution of Japanese space research. The essential mandate is "to conduct a small-scale mission about once every year, and a mid-to-large scale mission about once every five years by international collaborations."

This was a wise recommendation for Japanese space science. The Ministry of Education appreciated it and made their best effort to fulfill it. Even though one mission every year is not necessarily guaranteed, a predictable frequency of this order gives each major discipline (e.g., high-energy astronomy, magnetosphere, planetary, space engineering) a secure chance once every 4–5 years. Such continuity is a great privilege, allowing us to construct long-range plans while catching up to progress in each discipline. It is a system that works well, so much so that one of the members of a visiting committee, John Maddox, chief editor

of Nature at the time, praised ISAS in a story titled "Recipe for a Good Research Laboratory" (1993), in which he pointed out the benefits of autonomy and budgetary predictability in creating the framework for productive research.

### **Institute of Space and Astronautically Science Satellites: "Quick is Beautiful"**

One characteristic of the ISAS missions may be expressed as "small but frequent". ISAS satellites were small. What we could do was limited. It was therefore obvious that we should not base our research strategy on the same aims as that of big agencies. We focused our interests on well-chosen targets within the most up-to-date science. This was a restriction, but it was also a quick way to achieve limited goals. In contrast, a big mission is ambitious, attractive, capable, and powerful, but the opportunity for a big mission is infrequent. It requires a long time to build and is prone to delays.

The way ISAS conducts missions was appropriately expressed by Freeman Dyson's words, "Quick is beautiful." When he visited Minoru Oda, Director of ISAS, he admired the ISAS strategy by quoting the title of his own essay (Dyson 1986).

Another good reputation ISAS has is that the mission schedule is well maintained. Not only is the total budget fixed but also the funding period is limited to five years. Neither cost overruns nor delays are supposed to occur. This is, in practice, very difficult, but we maintained this standard, considering it our pride. In order to achieve this goal, simple design, tight planning, and self-regulation are required. Various tests, including environmental tests, were all conducted in-house and by our own responsibility. The virtue of doing so is that it not only saves costs but also trains scientists. We also attempted to minimize documentation. This is known to cut time and costs significantly.

Of course, reliability is of crucial importance in space missions. However, the assessment of reliability is not

straightforward, and there may be no golden rule. At the very least, I argued to minimize the number of black boxes. This also held when we accepted foreign instruments in cases of international cooperation. I insisted we form a team that included Japanese members so that they could get acquainted with the instrument through working together.

While participating in the integration and testing of a spacecraft every day, it is interesting to notice that a kind of love of the spacecraft is born. This is like nursing a baby. I think it is valuable, as it is a self-motivated, self-learning process. The scientists and engineers involved, including engineers/technicians from subcontractors, do their best not to overlook any irregularities without instructions. I believe that this enthusiastic attention contributes greatly to reliability.

I managed several missions after I moved to ISAS. I left technical details to the experienced engineers/technicians, rather than specifying them with documents or manuals. The work proceeded quietly but steadily without instructions from an inspector/supervisor. This went amazingly well. Foreign visitors were often impressed by the way the work was done so smoothly in silence. However, this must have looked scary to the professional managers. Once in a collaborative mission with the United States, a professional manager from NASA impatiently told me, "Yasuo, I am afraid your management is too sloppy. But I shall not interfere, since this is your mission." It may have appeared to him like an orchestra without a conductor. Yet, an excellent ensemble was maintained. Trusting the love of all the members was the way I did my job, and I believe it worked.

Needless to say, however, the above principles work for small missions. Luckily, the missions I was responsible for were all small. For example, the budget for ASCA (see below) was no more than US\$(2012)60 million, excluding the launcher and launch costs and the

international components. This was the biggest budget I ever had to deal with.

### **Moving to ISAS**

I moved to ISAS at the University of Tokyo in late 1974, when a new chair of space science was created. This happened to be just the middle point of my research life (till my retirement in 1994). I formed a new X-ray astronomy group with Masaru Matsuoka, Katsuji Koyama, and Hajime Inoue.

Soon after I came to ISAS, Oda told me, “You shall take management responsibility of a newly starting solar mission, Astro-A.” Mission management at ISAS involves (a) scientific planning of the mission, (b) proposal and budget application, (c) fabrication and tests, (d) schedule keeping, (e) in-orbit housekeeping, and (f) scientific observations. This is a lot. I asked Oda to give me a year of training. He told me bluntly, “This is one of the responsibilities of an ISAS professor.” This was a tough exercise. No one taught me how to manage satellite programs.

I spent 20 years at ISAS, in both its old and new incarnations. During this period, I was involved in seven different astronomy missions as either the mission manager or as an advisor/consultant. One mission typically takes seven years from the planning till launch, and the in-orbit operation time is added on top. Therefore, at any one time I was occupied with three or more different missions. This was confusing.

### **The Satellite Era**

With the first satellite launch in 1970, Japan entered into the satellite era. The first X-ray astronomy satellite, named CORSA (Cosmic Ray Satellite), came in 1976. However, the rocket failed to place the satellite into orbit due to a malfunction of the attitude control system. This was a great shock to all of us. Oda groaned saying, “This has caused a 10-year delay in Japanese X-ray astronomy.” But, he immediately challenged for a new chance. As a result of his extreme efforts, a chance for retrieval was given in 1979

(with CORSA-b). We made major modifications to the instruments.

During these years, X-ray astronomy advanced remarkably. The first X-ray satellite Uhuru went up in 1970, and discovered a great many Galactic and extragalactic X-ray sources.

The idea that luminous Galactic sources are mass-accreting compact binaries, including gravitationally collapsed objects—such as neutron stars or black holes—was widely accepted by the 1970s. (For its historical development, see Giacconi 2005). The binary nature was first established for Cyg X-1, even with convincing evidence from optical observations for the presence of a black hole (Webster & Murdin 1972, Bolton 1972). The seminal work on mass accretion was published by Shakura & Sunyaev (1973).

Furthermore, the high-sensitivity grazing-incidence X-ray telescope mission Einstein was soon to be launched. In these circumstances, what significant science could we do with our tiny satellite? Timely enough, a transient phenomenon called the X-ray burst of neutron stars was discovered (Grindlay & Heise 1975, Belian et al. 1976, Grindlay et al. 1976). A modulation collimator invented by Oda, which makes use of the satellite rotation, is capable of finding the position of the burst source within a wide field of view. Thus, the proportional counters equipped with the modulation collimator became the main instrument of CORSA-b.

### **CORSA-b “Hakucho”: 96 kg, Launched in February 1979**

This satellite was successfully launched in February 1979 and named Hakucho (Cygnus). The rotating modulation collimator caught eight new burst sources and pinpointed their locations. The X-ray bursts were interpreted as explosive burning of H<sub>1</sub> on the neutron star surface because it showed blackbody emission of about a 10-km radius. This phenomenon also excited

theoretical interests in Japan related to the equation of state of neutron stars.

### **Astro-A “Hinotori”: 188 kg, Launched in February 1981**

This was the first solar X-ray observation satellite from Japan. After the launch, it was renamed Hinotori (Phoenix). It participated in the International Solar Maximum Year (ISMY) Program (1979–81).

Hinotori was a small satellite, yet it yielded many new results about solar flares. It competed well with NASA's big satellite, the Solar Maximum Mission. The rotating modulation collimator viewed the entire solar disk and imaged solar flares above 20 keV for the first time. The Bragg spectrometer performed precision spectroscopy of the iron emission lines. In particular, the Fe +26 line confirmed production of over 30 MK plasmas in the flares. In addition, we prepared a small gas scintillation proportional counter, where the spectral resolution was better than an ordinary proportional counter (see the next section below). It measured wide-band X-ray spectra of solar flares, resolving the emission lines of various elements and the nonthermal component.

### **Astro-B “Tenma”: 216 kg, Launched in February 1983**

When I moved to ISAS in late 1974, I wondered what to do next. A grazing incidence telescope was out of the question. We had neither the technology nor the capacity. We looked for other themes that promised future development. Around that time, the mass accretion phenomenon on compact objects drew our attention. Detailed investigation of the X-ray spectrum of accretion disks was a fresh subject. In addition, there were many interesting objects for X-ray spectroscopy such as the supernova remnants (SNRs) and the clusters of galaxies, etc. So, we thought it worthwhile to put our effort into X-ray spectroscopy.

We started developing gas scintillation proportional counters (GSPCs) that provided energy resolution twice as good

as that of ordinary proportional counters (PCs), thus the GSPCs would be powerful for the study of spectral lines. A new graduate student, Takaya Ohashi, joined us on its development.

GSPCs require high tension of several kilovolts, much higher than ordinary PCs do, which makes it difficult to suppress the surface discharge. It took us quite a while to conquer the discharge with trial and error. We first made a small one for Hinotori and it worked well. Subsequently, we successfully produced bigger ones of 10 cm in diameter. We became confident when using them for the main instrument on Astro-B (Inoue et al. 1982).

The original manager of Astro-B was Oda. However, as soon as Hinotori became operational in orbit, he told me to take over responsibility for Astro-B. Because it was right after ISAS had been restructured and renamed, I could well understand Oda's heavy duty in administering ISAS. I had no choice other than to accept the management of Astro-B. Anyway, it carried the instruments we developed ourselves.

Astro-B was launched in February 1983 and renamed Tenma (Pegasus) (Tanaka et al. 1984). Its good energy resolution and wide energy range over a large-area (800 cm<sup>2</sup>) GSPCs were, as expected, advantageous. Some examples are given below. Regrettably, the battery system failed in less than two years, and the observing efficiency was severely limited after that.

Significant results were obtained from the emission lines and plasma diagnostics. The abundances of elements from the hot gas of SNRs and clusters of galaxies were investigated via emission lines. The 6.7-keV iron line was discovered in the Galactic plane (Koyama et al. 1986). Our advantage is being able to distinguish the 6.4-keV line (fluorescence) from the 6.7-keV line (thermal emission) of iron.

Most of the bright Galactic X-ray sources were known to be binaries of a low-mass

star and a neutron star, yet the nature of the X-ray spectrum was not yet clear. From a detailed study of the changes of the spectral shape with the X-ray intensity, Tenma made possible the important discovery that the spectrum was composed of two components (Mitsuda et al. 1984), a blackbody spectrum and another thermal spectrum. The former was identified with the emission from the neutron star surface and the latter with the emission from the accretion disk. The latter agrees with what Shakura & Sunyaev (1973) formulated. Mitsuda et al. (1984) further showed that the disk spectrum is described in terms of only two parameters: the radius of the innermost stable circular orbit (ISCO) and the innermost disk temperature. This model has now been widely accepted as the multicolor disk model (MCD) or disk blackbody model. It provides a basic model for the spectral analysis of X-ray binaries.

#### **Astro-C “Ginga”: 420 kg, Launched in February 1987**

Astro-C was the first large-scale international collaboration. It was the summer of 1980 when Prof. Ken Pounds of Leicester University and Dr. Harry Atkinson of the Science and Research Council (SRC) UK visited ISAS and proposed cooperation in space science. At that time, we were discussing the plan for a mission following Astro-B (still before the launch). This was later proposed as Astro-C. Naturally, cooperation on Astro-C became the central topic of the discussion. Later that year, Oda and I made a reciprocal visit to London to continue discussions on what the United Kingdom could offer.

The basic plan of Astro-C had already been decided early on. Its main instrument was to be a large area proportional counter array (LAC) of about half a square meter. (A GSPC was desirable, but it was difficult to make it so large). A small all-sky monitor and a small gamma-ray burst detector were later added.

The proposal of cooperation from the United Kingdom was that the Leicester University group would provide us with the LACs. For this proposal, all of the Japanese members (I recall there were no more than 20) got together and discussed whether or not to accept it. Opinions were divided. Some thought the UK group was more experienced in X-ray astronomy; hence we were afraid of losing leadership. Some argued to do it themselves, driven by the pride of experimental physicists. Had we voted, the collaboration could possibly have been rejected.

At that critical moment, Oda said definitively, “Tanaka, you are the person to decide it.” I grabbed the chance and declared, “Let us do the cooperation.” We also decided that Prof. Fumiyoshi Makino, who had newly come from Nagoya University, would take management responsibility of Astro-C and I would be responsible for the collaboration with Leicester University.

Thus, in early 1982, the Memorandum of Understanding for this collaboration was signed. This was an epoch-making start toward internationalization, and a historical step for Japanese space science.

The proportional counters made by the Leicester group were of excellent quality. The late Martin Turner and Takaya Ohashi (then at Leicester University) made crucial contributions toward its success. Yet, the collaboration was not completely without problems. Delivery was sometimes delayed, which worried us. Communication alone was not as easy as it is today. I often flew to the United Kingdom to fix problems. While we were waiting anxiously, the last proportional counter was delivered at the last minute!

Astro-C was launched in February 1987, and renamed Ginga (galaxy) (Makino 1987). Ginga will long be remembered among us because it completely wiped out the previously existing aversion to international collaboration. We realized that the merit of cooperation was much greater than the demerit. In fact, all the



later ISAS astronomical missions (at all wavelengths) have been conducted with international cooperation.

The US plan for the X-ray Timing Explorer (called the Rossi XTE or RXTE postlaunch), which was similar to Astro-C, had already existed in the early 1980s. However, details of all the US space science programs were withheld as a consequence of the Challenger accident. RXTE was finally launched in 1995. During that period, Ginga and the German Rosat (launched in 1990) were the only X-ray astronomy observatories. We offered a portion of observing time to NASA for US scientists.

Owing to its large photon collecting area and wide energy band coverage (2–30 keV), Ginga produced many results. In total, almost 1,000 papers have been published. I shall address three topics below.

Soon after the launch (on February 5, 1987), at midnight of February 24th, I received a surprising call: “A supernova went off in the Large Magellanic Cloud. Did Ginga see it?” This was SN1987A. We were amid the in-orbit checking. We obviously missed the first flash, but we decided to search for the hard X-rays coming out from the inside. This was not an easy task because the bright source Large Magellanic Cloud (LMC) X-1 was only half a degree away. By slowly scanning the region, we looked for any excess flux from the supernova. For the first five months, nothing abnormal was seen. But in August, we noticed the appearance of an unusually hard source and it brightened day by day. We were convinced of the emergence of hard X-rays from SN1987A and announced the discovery (Tanaka 1988).

The emergence of X-rays came much earlier than the theorists predicted and was the first evidence of large-scale mixing inside the debris. The hard X-ray instrument High Energy X-ray Experiment (HEXE) on board the Mir-Kvant later refined the Ginga result. Since this

discovery, the Rayleigh-Taylor instability became the standard exercise among supernova theorists.

Ken Pounds and his collaborators investigated the detailed spectra of Seyfert-type AGN by stacking 12 Ginga spectra (calling them Ginga-12). With thus enhanced statistics, they demonstrated the presence of a Compton hump around 20 keV. In addition, they showed the iron fluorescence line and an ionized absorber in the system. These results gave evidence for a Compton-thick disk and the reflection thereof, providing the basis for the accretion disk model for AGN.

The small all-sky monitor detected many transient sources, which were followed by detailed observations with the LAC. Among these transients, Ginga discovered four stellar-mass black-hole binaries (within about 20 currently known) confirmed by follow-up optical observations. Of these four, three showed soft thermal (MCD) spectra, and no blackbody component. (The fourth showed a power-law spectrum.) This is strong evidence that they are black holes, lacking the signature of a solid surface. The derived ISCO radii were indeed about 3 Schwarzschild radii for the black holes of the optically determined mass (Tanaka & Lewin 1995).

Later, the accuracy of optical measurements improved dramatically, allowing precise determination of the ISCO radius. The measured mass and the ISCO radius led to the estimation of the spins of black holes (Remillard & McClintock 2006).

### **Solar-A “Yohkoh”: 390 kg, Launched in August 1991**

I recall that the discussion on Solar-A started around 1983. In the beginning, a group of cosmic-ray physicists took the initiative in the planning. One day, the late Katsuo Tanaka, who had taken leadership of Hinotori, came to me and said that the current plan was not what solar physicists considered to be most important.

Though it was a little harsh, I requested to reset the existing plan and proposed to hold a small international workshop for solar physics experts. This was to discuss, given Solar-A's capacity, what could be the most valuable experiments. As a result of this meeting in 1985, they concluded that a combination of a soft X-ray imager (SXI) and a hard X-ray imager (HXI) would be most productive.

The SXI is a wide-field grazing incidence telescope. The basic plan was to work in cooperation with the United States. I visited NASA Headquarters with Yoshiaki Ogawara, who had been appointed as the Solar-A mission manager. Director of Astrophysics at NASA Dr. Charles Pellerin was supportive, but the associate administrator did not show interest. At that time, the flagship projects prevailed over the Division. (These

were Hubble, Compton Gamma Ray Observatory, Chandra, and Spitzer. None had yet been launched.) Clearly, he had no interest in such a small collaboration. I said to him, "This proposal is small, but it promises to do important science. If you are not interested in such a small [collaboration], there [will] be no future cooperation with ISAS." The associate administrator was flustered and told the Director to go ahead with the collaboration. This was how space science cooperation between ISAS and NASA started. Since then, an intimate relationship with NASA has continued to this day.

The soft X-ray telescope mirror was prepared by the Lockheed group. This instrument was the SXI. The HXI was a sophisticated Fourier synthesis X-ray telescope. Kazuo Makishima's group completed it. Oda called it "the ultimate modulation collimator." There was also minor UK participation with a small UV spectrometer. Solar-A was launched in August 1991 and renamed Yohkoh (sunbeam). Its life covered a complete 11-year cycle.

According to the lead scientist Saku Tsuneta, the most important result

of Yohkoh was that it established the origin of solar flares to be the reconnection of magnetic loops. The frequently observed jet phenomena were also shown to result from reconnections. Furthermore, the observations revealed numerous microflares, and these microflares were found to be major heat sources of the corona.

HXI frequently observed "twin-eyed" hard X-ray sources. These were interpreted to be the chromospheric hot spots produced by particles accelerated at the magnetic loop top and flowed down along the fields. HXI also found hard X-ray hot spots at the loop tops, which were most probably the reconnection sites.

### **Astro-D "ASCA": 420 kg, Launched in February 1993**

We had started planning Astro-D around 1985, even before the launch of Ginga. In the United States, the big grazing incidence telescope Advanced X-ray Astrophysics Facility (AXAF) was in preparation. We were more than aware of its focusing and imaging power. We regretted that we had neither the mirror technology nor the capacity to carry the weight. Despite these restrictions, we once had a study contract of an X-ray telescope satellite with Perkin-Elmer for a possible future mission. However, this was not realized.

It came to my mind that there was one possibility of realizing a telescope mission. That was the foil mirror technology that Peter Serlemitsos of NASA's Goddard Space Flight Center had developed over a number of years. The telescope uses multinested conical foils that fill the aperture with reflecting surface, enabling a light-weight large-effective-area X-ray telescope. The image resolution is modest, but it can cover a wide energy range. Its greatest merit was its superlight weight, well suited for Astro-D. Interestingly, it had never been flown on a US satellite [except for the Broad Band X-ray Telescope (BBXRT) on the space shuttle Columbia]. In response to our contact,

NASA was in full support of this collaboration. At that time, Hideyo Kunieda of Nagoya University was with Serlemitsos engaged in the development of it with him. This made cooperation a lot easier.

However, a difficult problem existed. The focal length of the telescope was 3.5 m, too long to be housed within the rocket nose fairing. I had an idea for an extensible optical bench: keeping it short during the launch and extending it in orbit. Some people expressed serious concern, which was understandable. Although I was confident, I consulted with a mechanics specialist. He tested the reliability by constructing a mechanical model, and after careful evaluation gave me a “Go” sign! An extensible optical bench was installed on Astro-D for the first time carrying four of the thin foil telescopes.

As regards the focal plane instruments, there was a timely offer from George Ricker of MIT. His group had been developing the X-ray CCD for AXAF. Because AXAF had been delayed, NASA supported releasing the CCD camera for use on Astro-D. The CCD had energy resolution an order of magnitude higher than proportional counters, which was a big advantage for line spectroscopy. In this sense, Astro-D benefited from the AXAF program. Of four telescopes, two foci were equipped with these CCDs, and the other two were equipped with the imaging gas scintillation proportional counters (IGSPC) prepared by Makishima's group. Ohashi played key roles in the fabrication of them. All four detectors showed excellent performance.

The team was quite international. I took management responsibility. Hajime Inoue assisted me as the deputy manager. Steve Holt, science director of the GSFC at that time, became the principal investigator for the US side. We also formed the Astro-D Science Working Group (SWG) to discuss the scientific issues. In addition, consulting with NASA Headquarters, several senior scientists—Claude

Canizares, David Helfand, Richard Mushotzky, Dan McCammon, and John Nousek from the United States and Andy Fabian from the United Kingdom—were invited as science advisors. They worked jointly with key Japanese scientists. Many more US scientists participated in either hardware/software preparation or scientific discussions.

Astro-D was launched in February 1993 and renamed ASCA (Advanced Satellite for Cosmology and Astrophysics) (Tanaka et al. 1994). The pronunciation of ASCA mimics the name of a legendary bird (“asuka”) as well as that of a culturally flourishing era in the Japanese history.

Observing time through ASCA was not only shared with NASA but also offered to the ESA. In this way, ASCA was utilized internationally. It would not be an exaggeration to say that ASCA together with Rosat sustained X-ray astronomy in the 1990s. These two observatories had complementary capabilities; hence they made a good combination. Rosat had superior image resolution and high sensitivity, whereas ASCA had wide energy range coverage and superior spectral resolution. Some significant results from ASCA are listed below.

The red ward broadening of the iron line was detected for the first time in MCG 6-30-15, which can be ascribed to a general relativistic effect of strong gravity close to black holes (Tanaka et al. 1995). This effect had been predicted by Andy Fabian (1989), an ASCA team member, and in fact it was he, motivated by some unknown inspiration, who insisted that this specific Seyfert 1 galaxy be observed. Since then, many more red ward-broadened iron lines were found, not only for AGN but also for Galactic black holes. The study has eventually been extended to detect the spin of black holes.

Toward the Galactic Center, ASCA discovered strong iron fluorescent lines from the molecular clouds surrounding the Center. This was interpreted to be the result of X-ray irradiation that had taken

place several hundred years ago, when the Center had been active (Koyama et al. 1996).

ASCA also provided insight on the CXRB. It was, by that time, believed to be produced by numerous AGN. The problem then was that the CXRB spectrum differed from the superposition of the observed power-law spectrum of bright AGN. The contribution from the populous presence of heavily absorbed AGN (type 2 AGN) was suspected. ASCA, which possessed the highest sensitivity in the 2–10 keV band uncovered many such heavily absorbed AGNs.

ASCA not only enriched our knowledge of clusters of galaxies and supernova remnants but also of the supernova itself, which occurred soon after the launch. I refer to supernova SN1993J in the nearby galaxy M81. That reminded us of SN1987A right after the Ginga launch.

#### **More Astronomy Missions of ISAS**

After my retirement in 1994, the following astronomy missions (all launched by M-V rockets) have been conducted:

- The Highly Advanced Laboratory for Communications and Astronomy (HALCA) “MUSES-B” was the first space very long baseline interferometry experiment, launched in 1997.
- Astro-EII “Suzaku,” an X-ray astronomy mission following ASCA, was launched in 2005 and is still operational.
- An infrared mission Astro-F “Akari,” launched in 2006, completed its sky survey from near- to far-IR with an order of magnitude higher sensitivity than IRAS.
- Solar-B “Hinode” (sunrise) is the third solar mission following Yohkoh. It was launched in 2006 and is currently operational.

More detailed descriptions of each individual mission can be found at the following

website: <http://www.isas.jaxa.jp/e/index.shtml>.

#### **X-RAY Astronomy Community**

When X-ray astronomy took shape in Japan around 1966, there were only two groups, one in Nagoya University (U-lab of Hayakawa) and the other at ISAS (Oda's lab), and there were altogether no more than 10 members. Even in mid-1980, the total number including graduate students was 30 or so. The lack of manpower quickly became serious as we entered into the satellite era.

Operation of a satellite requires considerable manpower for various tasks; to monitor the satellite health, to conduct a sequence of observations, to take a quick look at data, to assist guest observers, etc. There are jobs that can be subcontracted, but the commitment of scientists is indispensable. Also, it is an important aspect that these works provide scientists with valuable opportunities for learning how satellite observations are conducted.

The ISAS satellite capacity doubled in about five years. Scientists became able to conduct many more observations with much more sophisticated instruments. This required that the commitment of scientists increase steadily. The community of scientists (including international participation) also needed to be of a proper size for operating satellites and harvesting scientific results.

I made an effort to enlarge the X-ray astronomy community. The first opportunity was at Osaka University. My friend, then the Dean of the Faculty of Science, had been interested in this quickly developing field. I succeeded in convincing him to open a new X-ray astronomy group in Osaka. Thus Shigenori Miyamoto was invited to be a professor there. Successively, new groups were born in Riken (Masaru Matsuoka), the University of Tokyo (Kazuo Makishima), Kyoto University (Katsuji Koyama), and so on. It was obvious that the success of our missions such as Tenma and Ginga attracted them. The

Japanese X-ray astronomy community has steadily grown. As of 2010, the number of community members possibly exceeds 100.

There is, however, a remarkable fact. None of these new groups is in an astronomy department. They all belong to physics departments. It is true that there are only a few astronomy departments in Japan. Yet, this is a bit unusual. X-ray astronomy is obviously a mature astronomy discipline; hence it should be integrated into the astronomy course of education. It is important that X-ray astronomy students learn a full course of astronomy. The gap between the two communities is a loss to both. I did not receive an astronomy education. I feel it painfully as it gave me a significant handicap.

### **Reorganizations of ISAS**

ISAS was further radically changed in 2003. It was a merger of three space-related organizations: ISAS, NASDA (National Space Development Agency), and NAL (National Aerospace Laboratory). This merger was forced as a consequence of the reform of the central government in 2001. The Science and Technology Agency that supervised the latter two space organizations was merged with the Ministry of Education, which hosted ISAS. The officials thought that the activities of these organizations were redundant without pondering further what these institutes had been doing. ISAS is the only institute that carries out academic research and is the smallest among the three.

The merit of this merger for ISAS may, in principle, be to enable larger missions with NASDA's powerful rocket. However, opportunities are rare because of the high cost, and risk increases with the complexity of the mission.

In my opinion, "small but frequent" was the distinguishing feature of the ISAS missions that have made the world community appreciate Japanese space science and its X-ray astronomy. ISAS should not lose this unique feature. In this

respect, many scientists regret that the M-V rocket developed by ISAS was abandoned as a result of the merger. It was too hasty. A new solid-propellant rocket named Epsilon that could substitute M-V for medium-class missions is now under development. I strongly hope that the virtues of ISAS will be maintained in the future.

The basic culture was different between ISAS and the other two organizations. ISAS used to conduct small-to-medium-sized missions, where researchers were trained to judge based on their own experiences, whereas NASDA's aims were to develop big boosters and applications satellites, and to further develop the manned space program through participation in the International Space Station. For such big programs, strict application of detailed specs is given high priority. So, the NASDA members' skills may be best used to judge things relying upon documents rather than with their insufficient hands-on experience. I seriously hope that good understanding and cooperation are achieved between the organizations.

Academic freedom and interuniversity autonomy are invaluable assets of ISAS to carry on. It will be the most important obligation of the present generation to maintain the excellence ISAS achieved in the past.

### **Closing**

I retired from ISAS in March 1994 after enjoying fruitful results from ASCA for one year. I should say that my last year at ISAS was the richest year in my research life. My group became truly international. This environment was exactly what I dreamed of during my time in Leiden.

When I look back, my research life coincided with the time of "Sturm und Drang" in astronomy. I have not only witnessed the exploding growth of X-ray astronomy from its birth but also been lucky enough to take part in its development. This was possible through my drift from studying cosmic rays into

studying cosmic X-rays at the right time. When I first entered research work, I never imagined I'd go into astronomy. My research life was an unscheduled journey.

### **A Literature Review**

The problem of powering active galactic nuclei, X-ray binaries and quasars is one of the most important problems today in high energy astrophysics.

Several mechanisms have been proposed by various authors (Abramowicz, Calvani & Nobili 1983; Rees *et al.*, 1982; Koztowski, Jaroszynski & Abramowicz 1978; Shakura & Sunyaev 1973; for an excellent review see Pringle 1981).

**Rees et al. (1982)**, argue that the electromagnetic extraction of black hole's rotational energy can be achieved by appropriately putting charged particles in negative energy orbits.

**Blandford & Znajek (1977)** have also proposed an interesting mechanism by considering the electron-positron pair production in the vicinity of a rotating black hole sitting in a strong magnetic field. It is, therefore, important to study the energetic of a black hole in electromagnetic field.

**Penrose (1969)**, an ingenious and novel suggestion for the extraction of energy from a rotating black hole. It is termed as the Penrose process and is based on the existence of negative energy orbits in the ergo sphere, the region bounded by the horizon and the static surface.

**Vishveshwara (1968)**, though there does not exist an ergo sphere for the Reissner-Nordström black hole, there do exist negative energy states for charged particles.

**Denardo & Ruffini (1973)**, which means that the electromagnetic energy can also be extracted by the Penrose process.

**Penrose (1969)**, Though did not consider astrophysical applications of the and others proposed that it could provide a viable mechanism for high energy jets emanating from active galactic nuclei.

The mechanism envisaged a star-like body which on grazing a super massive black

hole breaks up into fragments due to enormous tidal forces

**(Mashhoon 1973, Fishbone 1973)**. Some fragments may have negative energy orbits and they fall into the black hole resulting in reduction of its rotational energy while the others come out with very high velocities to form a jet. However, this process fell out of favor for its astrophysical applications owing to limits on the relative velocity between the fragments

**(Bardeen, Press & Teukolsky 1972, Wald 1974)**: No significant gain in energy results for an astro physically reasonable orbit of an incident star unless the split up itself is relativistic, *i.e.* relative velocity between the fragments  $1/2$ . Very recently, **Wagh, Dhurandhar & Dadhich (1985)** have shown that these limits can be removed with the introduction of an electromagnetic field around the black hole. The electromagnetic binding energy offers an additional parameter which is responsible for removal of the limits. Thus the Penrose process is revived as a mechanism for high energy sources.

In this paper we wish to study the negative energy states for charged particles in the Kerr-Newman space time with a view to extracting energy by the Penrose process.

**Prasanna (1983)**, A comparative analysis of negative energy states for charged particles in the Kerr-Newman field and for a Kerr black hole in a dipole magnetic field is done by We study the negative energy states in a greater detail, and set up a Penrose process for energy extraction and also examine its efficiency in this case. It is known (**Chandrasekhar 1983**) that the maximum efficiency of this process is 20.7 per cent in the case of a Kerr black hole. The presence of charge on the Kerr-Newman black hole decreases the efficiency further when uncharged particles participate in the process while the efficiency is enormously enhanced (as high as over 100 per cent, in fact there is no limit!) when charged particles are involved.

Astrophysical massive bodies are not known to have significant charge on them [ $Q/(\sqrt{GM}) \ll 1$ ]. That means the charge  $Q$  on the black hole should be taken as very small. But a small, nonzero  $Q$  can have appreciable effect on the test charge orbits due to the Lorentz force. It is the Coulombic binding energy that contributes significantly to the energy of the test particle. It is not unjustified, therefore, to study the Penrose process with this assumption.

## 2. The Kerr-Newman field

The Kerr-Newman space time in the Boyer Lindquist coordinates is described by the metric

$$Ds^2 = -(\Delta/\rho^2)(dt - a \sin\theta d\phi)^2 + (\rho^2/\Delta)dr^2 + \rho d\theta^2 + (\sin^2\theta/\rho^2)[(r^2 + a^2)d\phi - a dt]^2 \quad (2.1)$$

Where 
$$\Delta = r^2 + a^2 - 2mr + Q^2$$

$$\rho^2 = r^2 + a^2 \cos^2\theta$$

Here  $m$  is the mass,  $a$  is the angular momentum per unit mass and  $Q$  is the charge on the black hole. We have used the geometrised units ( $c = 1, G = 1$ ). The event horizon is given by the larger root  $r_+$  of  $\Delta = 0, r_+ = M + (M^2 - a^2 - Q^2)^{1/2}$

In this space time there exists an electromagnetic field due to the presence of charge  $Q$ .

This field is obtained from the vector potential  $A_i$ ,

$$A_i = (-Qr/\rho^2, 0, 0, aQr \sin^2\theta/\rho^2) \quad (2.2)$$

That means the rotation of the black hole also gives rise to a magnetic dipole potential in addition to the usual electrostatic potential.

### 2.1 The Equations of Motion

Let a test particle of rest mass  $\mu$  and electric charge  $e$  move in the exterior field of the black hole. Its motion will be governed by the gravitational field of a charged rotating black hole as well as by the Lorentz force due to electromagnetic interaction. The equations of motion of the particle can be derived either from the Lagrangian

$$\mathcal{L} = \mu/2 g_{ij} \dot{x}^i \dot{x}^j + e A_i \dot{x}^i \quad (2.3)$$

or from the Hamiltonian  $H$ ,

$$H = \frac{1}{2} g^{ij} p_i p_j \quad (2.4)$$

Where a dot denotes derivative with respect to the affine parameter  $\tau/\mu$  ( $\tau$  being the proper time) and  $p_i$  is 4-momentum of the particle. Since the metric and the electromagnetic field are time independent and axially symmetric, the energy and the  $\phi$  component of the angular momentum will be conserved yielding two constants of motion. Carter (1968), Showed that the Hamilton Jacobi equation is separable in this system giving the constant related to the  $\theta$ -motion of the particle. It is known as the Carter constant (Misner), have

$$\partial \mathcal{L} / \partial t = p_t + e A_t = -\mu E \quad (2.5)$$

$$\partial \mathcal{L} / \partial \phi = p_\phi + e A_\phi = \mu L \quad (2.6)$$

Where  $E$  and  $L$  are the energy and the  $\phi$ -component of the angular momentum per unit rest mass of the particle as measured by an observer at infinity.

The rest mass  $\mu$  of the particle gives another first integral

$$-\mu^2 = g^{ij} p_i p_j \quad (2.7)$$

Now, on substituting Equations (2.5) and (2.6) in (2.7)

We obtain

$$g_{\phi\phi}(E + eA_\phi)^2 + 2g_{t\phi}(E + eA_\phi)(L - eA_\phi) + g_{tt}(L - eA_\phi)^2 + \psi(g^{rr}p_r^2 + g^{\theta\theta}p_\theta^2 + \mu^2) = 0 \quad (2.8)$$

Which gives?

$$E = -eA_\phi + \omega(L - eA_\phi) + (\sqrt{-\psi/g_{\phi\phi}})[(L - eA_\phi)^2 + g_{\phi\phi}(g^{rr}p_r^2 + g^{\theta\theta}p_\theta^2 + \mu^2)]^{1/2}, \quad (2.9)$$

$$\Psi = g_{ii} g_{\phi\phi} - g_{i\phi}^2 < 0$$

For  $r > r_+$

$$\omega = -g_{t\phi}/g_{\phi\phi} > 0$$

The event horizon  $r_+$  is given by the larger root of  $\Psi = 0$ . It can be easily verified that  $\Psi = 0, \Delta = 0$ .

For convenience we introduce the dimensionless quantities

$$R = r/m, a = a/m, l = L/m, Q = Q/m \quad (2.10)$$

$$A_i = A_i/m, A_\phi = A_\phi/m, e = e/\mu$$

And drop the bars on these symbols in further discussion.

### 2.2 The Effective Potential

By the symmetry of the metric and the electromagnetic field, it follows that the particle commencing its motion with  $p_\theta = 0$  in the equatorial plane will stay in the plane for all time, *i.e.*  $p_\theta = 0$  all through

the motion. This can also be verified by considering the equations of motion

$$x^{-i} + \Gamma^i_{ki} x^{ik} x^{-i} = eF^i_{kx} x^{-k} \dots \dots \dots (2.11)$$

For the  $\theta$ -coordinate. The Lorentz force term on the right gives a force directed in the  $\theta = \pi/2$  plane for  $A_i$  given in Equation (2.2) and  $F_{ik} = A_{k,i} - A_{i,k}$ . Henceforth we shall consider motion in the equatorial plane and set  $p_\theta = 0$ . As our main aim in this investigation is to analyse negative energy states, the restriction of motion in the equatorial plane will not matter much.

The effective potential for radial motion could be obtained by putting  $pr = p_\theta = 0$  in Equation (2.9).

We write

$$V = -eA_i + \omega(L - eA_\phi) + (\sqrt{-\psi/g_{\phi\phi}})[(L - eA_\phi)^2 + g_{\phi\phi}]^{1/2} \dots \dots \dots (2.12)$$

$$\omega = -g_{i\phi}/g_{\phi\phi} > 0$$

The positive sign for the radical is chosen to ensure that the 4-momentum of the particle is future directed. The quantity  $\omega$  represents the angular velocity of a locally non rotating observer (LNRO) at a given  $r$  and  $\theta$ . That is, a particle with  $L = 0$  will have  $d\phi/dt = \omega \neq 0$ .

Equations (2.8) and (2.12) can be rewritten as

$$\alpha E^2 - 2\beta E + \gamma = 0 \dots \dots \dots (2.13)$$

$$V = \beta + (\beta^2 - \alpha\gamma)^{1/2}/\alpha \dots \dots \dots (2.14)$$

Where

$$\begin{aligned} \alpha &= (r^2 + a^2) - \Delta a^2, \\ \beta &= (r^2 + a^2)(a\lambda + eQr) - \Delta a\lambda, \\ \gamma &= 9a\lambda + eQr)^2 - (r^2 + l^2) \dots \dots \dots (2.15) \end{aligned}$$

The effective potential at the horizon reads as

$$V(r_+) = eQ/r_+ + a/(r_+^2 + a^2) (1 - eQa/r_+) \dots \dots (2.16)$$

Where

$$\omega(r_+) = a/(r_+^2 + a^2)$$

$V(r_+)$  can become negative if  $-eA_t < 0$  and  $(l - eA_\phi) < 0$ . It should be noted that it is the sign of  $(l - eA)$  which is relevant for  $V$  getting negative (Dadhich 1983). The particle rotates slower than the LNRO if  $l - eA < 0$ . This can be seen from the following.

The angular velocity  $\Omega = d\phi/dt$  of a particle can be obtained by using Equations (2.5) and (2.6),

$$\Omega - \omega = (-\psi/g_{\phi\phi}^2)(1 - eA_\phi)[E + eA_t - \omega(1 - eA_\phi)]^{-1}$$

Which, in view of Equations (2.9) and (2.10), directly relates the sign of  $(l - eA_\phi)$  to  $\Omega - \omega$

As argued by Dadhich (1985),  $\Omega - \omega > 0$  defines co/counter-rotation relative to an LNRO. It is the LNRO frame that is physically meaningful in these considerations.

### 3- THE NEGATIVE ENERGY STATES

In this section we shall discuss the behavior of the effective potential in relation to the occurrence of negative energy states (NES). The NES could occur due to the electromagnetic interaction (as in the Reissner Nordström case) as well as due to the counter-rotating orbits (as in the Kerr case). The Kerr-Newman solution represents the gravitational field of a charged and rotating black hole. The rotation of a black hole also gives rise to the magnetic dipole field in addition to the usual electrostatic field. The presence of electromagnetic field will favor the occurrence of NES (Dhurandhar & Dadhich 1984 a, b) (i) by allowing larger negative values for energy, and (ii) by increasing the region of occurrence of NES. It is also known to cause in certain situations the splitting of NES region into two disjoint patches (Dhurandhar & Dadhich 1984a, b). However, in the Kerr-Newman field it turns out that NES may occur only in one patch extending upto the horizon (Prasanna 1983) as in the Kerr case. In the following we shall investigate NES with reference to counter rotation and electromagnetic interaction.

#### 3.1- THE EFFECTIVE POTENTIAL CURVES

Let us first look at some typical plots of the effective potential which exhibit its dependence on the parameters  $l$  and  $\lambda = eQ$ .

##### 3.1.1- THE SINGLE-BAND NES STRUCTURE

The single band NES structure as also noted by Prasanna (1983), we establish this character analytically.



From Equations (2.13) and (2.14),  $V = 0$  requires  $\gamma = 0$  and  $\beta < 0$ . From

Equation (2.15)  $\gamma = 0$  gives  
 $(al + eQr)2 - \Delta(r2 + l2) = 0 \dots\dots\dots(3.1)$

We now show that there is only one root for the above equation for  $r > r+ = 1 + (-1)$ .

The effective potential  $V$  is plotted for  $a = 0.8$  and  $Q = 0.5$ . The vertical axis is drawn at the horizon ( $r+ = 1.33$ ). (a)  $l$  takes the values  $-100, -50, -10, 0, 5$ ; (b)  $\lambda$  ranges through  $-10, -5, -2, 0, 5$ . The curve corresponding to a particular value of  $l$  and a particular value of  $\lambda$  can be picked up from the property that  $V(r+)$  is a monotonically increasing function of  $l$  and  $\lambda$ .

$a2 - Q2)1/2$ . Write  $R = r - r+$ . The above equation then reads as  
 $R^4 + AR^3 + BR^2 + CR + D = 0 \dots\dots\dots(3.2)$

Where  
 $A = 2(1 + 2\sqrt{1 - a^2 - Q^2})$   
 $B = 5r_+^2 - 4r_+ + l^2 - \lambda^2$   
 $C = 2[r_+^3 - r_+^2 + r_+(l^2 - \lambda^2) - l^2 - a\lambda]$   
 $D = -(al + eQr_+)^2$

To establish the result we apply Descartes' rule of signs. As  $A > 0$  and  $D < 0$ , the above equation can have more than one positive root only when  $B < 0$  and  $C > 0$ . We now show that this is not possible.

Let  $B < 0$ , which implies  
 $\lambda^2 > 5r_+^2 - 4r_+ + l^2 \dots\dots\dots(3.3)$

Which makes  
 $C < -2(4r_+^3 - 3r_+^2 + l^2 + a\lambda)$

If  $l\lambda > 0$ , then  $C < 0$ . However, for  $l\lambda < 0$ ,  $C < 0$  will require  
 $4r_+^3 - 3r_+^2 + l^2 > |a\lambda|$

Squaring both sides of the above inequality and using (3.3) we deduce  $C < 0$  for this case too. This proves the result. Thus  $\gamma = 0$  has only one root  $r > r+$ . As  $r \rightarrow \infty$ ,  $V \rightarrow l$ , and hence the NES band will occur only when  $V < 0$  at the horizon.

The single-band nature of NES prescribes a linear relationship between  $l$  and  $\lambda$ , which could be inferred from  $V(r) < 0$ . From Equation (2.16) this will imply,  
 $l < -\lambda r + /a$ .

**3.1.2- THE EXTENT OF THE NES BAND**

To find the extent of the NES band we consider the quadratic Equation (3.1) in various limits as the exact solution is not easily obtainable. We take  $|l| \ll 1$  and  $|\lambda| \ll 1$  in  $V = 0$  for larger  $r$ . Then the quadratic reduces to a cubic

$R^3 - 2r^2 + (a^2 + Q^2 + l^2 - \lambda^2)r - 2l(a\lambda + l) = 0$   
 By dropping  $Q^2$  terms as  $Q^2 > l^2 \dots\dots\dots(3.4)$

Case (i): Let  $|\lambda| \sim |l|$ ,  $l(a\lambda + l) > 0$ . For large  $r$ , terms in  $r^2$  and  $r$  can be neglected implying  
 $r \approx [2l(a\lambda + l)]^{1/3}$

Case (ii):  $|l - \lambda| \ll 1$ , then Equation (3.4) reduces to  
 $R^2 \approx (l^2 - \lambda^2 + a^2 + Q^2)$

by neglecting  $r^2$  and the constant terms. Since,  $a, Q < 1$  and if  $l^2 - \lambda^2 > 0$  then in the general case we need to resort to numerical computations. Table 1 below gives the extent of NES. It gives the root of  $V = 0$  for various values of  $l$  and  $\lambda$  for fixed  $a (= 0.8)$  and  $Q (= 0.5)$ . The horizon in this case is at  $r+ = 1.3317$ .

It is apparent from the table that for a fixed  $\lambda < 0$ , the value of  $r = r0$ , say, where  $V$  gets negative, increases as  $l$  increases until  $l$  becomes positive and dominant, then it drops off below  $r+$ . On the other hand, for  $\lambda > 0$ ,  $r0$  decreases as  $l$  increases and less negative but it slightly increases for  $|\lambda|$  small and then steadily decreases as  $X$  increases further in the positive range. For  $l > 0$ , only large negative values of  $\lambda$  give  $r0 > r+$ . The large negative  $\lambda$  favors large values for  $r0$ , as is borne out by the special cases discussed

**3.1.3- THE FACTORS CAUSING NES**

From Equation (2.12) it is seen that  $V$  can be negative only when  $\lambda = eQ < 0$  (i.e.  $eA\phi < 0$ ) and/or  $(1 - eA\phi) < 0$ . Here we wish to compare the contributions of these factors in rendering  $V < 0$ . There are the following six possible cases.

- (1)  $-eA_i < 0$   $-eA_\phi > 0$   
 $l > 0$ ,
- (2)  $-eA_i < 0$   $-eA_\phi > 0$   
 $l < 0$ ,
- (3)  $-eA_i < 0$   $-eA_\phi < 0$   
 $l > 0$ ,

- (4)  $-eA_i < 0$   $-eA_\phi < 0$   
 $l < 0$ ,  
(5)  $-eA_i > 0$   $(1 - eA_\phi) < 0$   
 $l > 0$ ,  
(6)  $-eA_i > 0$   $(1 - eA_\phi) < 0$   
 $l < 0$ ,

One can immediately see that case (3) is not possible because the conditions put on the parameters are inconsistent in view of Equation (2.2). That is,  $\lambda < 0$  and  $l > 0$  do not permit counter-rotating orbit ( $\Omega - \omega < 0$ ).

The second law of the black hole physics rules out case 5. It implies MTW),

$$\Delta m > (a \delta J + Q \delta Q r_+) / 2r_+,$$

Where:

$$\delta m = \mu E, \delta J = \mu m l, \delta Q = e \mu.$$

Clearly  $e > 0$ , and  $l > 0$  does not allow  $\delta m < 0$ , thus ruling out NES. That is, the magnetic field alone cannot make  $V < 0$ .

The rest of the four cases allow for the NES. In the first case, the electrostatic energy is responsible for the NES while in case 2 it is the electrostatic and rotation, in case 6 the rotation and magnetic field, whereas in case 4 all the three factors join hands.

We shall consider the cases 1, 2 and 6 for  $Q \rightarrow 0$  but  $\lambda = eQ$  finite.

From Equation (2.16),  $V(r_+) < 0$  gives

$$\lambda / r_+ + \omega (1 - a \lambda / r_+) < 0$$

Where by neglecting  $Q^2$ . Then

$$L < -\lambda (2/a - a/r_+) \dots \dots \dots (3.5)$$

In case 1, the inequality (3.5) gives

$$l - \lambda < 2/a - a/r_+$$

Which, in the extreme case  $a \rightarrow 1$ , implies  $l < |\lambda|$ . In case 2, it will always be satisfied, while in case 6 it gives

$$l / \lambda < a/r_+ - 2/a,$$

Which will imply for  $a \rightarrow 1$ ,  $|l| > \lambda$ .

#### 4- THE ENERGY EXTRACTION

In this section we consider the process of energy extraction from the black hole. In this process proposed by Penrose (1969), it is envisaged that a particle falling onto a black hole splits up into two fragments at some  $r > r_+$  where  $V < 0$ . Then, if one of the fragments has negative energy (relative to infinity), it will be absorbed by the black hole while the other fragment will

come out, by conservation of energy, with the energy greater than the parent particle. This will result in extraction of energy from the black hole. In the case of the Kerr-Newman black hole, the extracted energy may be provided by the rotational and/or the electromagnetic energy (Christodoulou 1970). In the following we shall first consider the conservation equations for the 4-momenta of the Participating particles and then give a recipe for energy extraction. At the point of split, we assume that the 4-momentum is conserved, i.e.

$$P_1 = P_2 + P_3 \dots \dots \dots (4.1)$$

Where,  $p_i$  ( $i = 1, 2, 3$ ) denotes the 4-momentum of the  $i$ th particle. The above relation stands for the following three relations.

$$E_1 = \mu_2 E_2 + \mu_3 E_3 \dots \dots \dots (4.2)$$

$$L_1 = \mu_2 l_2 + \mu_3 l_3 \dots \dots \dots (4.3)$$

$$T_1 = \mu_2 t_2 + \mu_3 t_3 \dots \dots \dots (4.4)$$

Where, we have set  $\mu_1 = 1$ . The other conservation relation follows from the conservation of charge,

$$\lambda_1 = \mu_2 \lambda_2 + \mu_3 \lambda_3 \dots \dots \dots (4.5)$$

The quantities  $\mu_i, l_i, \lambda_i, E_i, r_i$  refer to the  $i$ th particle. These relations contain in all eleven parameters, of which 7 can be chosen freely. The choice of these parameters will be constrained by the requirements that particle 1 should reach the point of split where  $V < 0$  for some suitable  $l, \lambda$  values such that particle 2 can have  $E_2 < 0$  and particle 3 has an escape orbit. To ensure uninterrupted progress of particle 1 down to the horizon,

We set  $l_1 = 0 = \lambda_1$ . The  $l$  and  $\lambda$  parameters for particle 2 should be so chosen that  $E_2 < 0$ . We further chose  $r_2 = 0$  which will imply  $E_2 = V$  at the point of split. Such a choice is favorable for high efficiency of the process. (For further discussion refer to Dhurandhar & Dadhich 1984b.)

For these calculations we assume  $Q \ll 1$ . This assumption is realistic as can be seen from the following relation

$$Q \text{ (meters)} = (G / \epsilon_0 c A)^{1/2} Q \text{ (Coulombs)}.$$

Though  $Q$  could be small,  $eQ = \lambda$  can produce the Lorentz force on a particle of

the charge/mass ratio of an electron, comparable to the corresponding gravitational force. So we neglect  $Q$  in the metric but retain  $\lambda$  in the equations of motion.

We shall now adopt the scheme for calculations due to Parthasarathy *et al.* (1985). From Equation (2.8) we can readily write the equations for radial motion of the particle,

$$R^2 = 1/r^3 [RE^2 - 4aEL - 9r - 2)L^2 - r\Delta] \dots \dots \dots (4.6)$$

*energetic of Kerr-Newman black hole 95*

Where

$$R = r(r^2 + a^2) + 2a^2,$$

$$E = E + eA_i,$$

$$L = l - eA_\phi,$$

Since we have taken  $r_2 = 0$ , which means

$$R_1 = \mu_3 r_3 \dots \dots \dots (4.7)$$

By writing Equation (4.6) for particles 1 and 3 and using Equations (4.2), (4.3), (4.5) and (4.7) we obtain  $E_1$  as follows

$$E_1 = [RE_2^2 - 4aE_2L_2 - (r-2)L_2^2] \mu_2^2 + r\Delta(1 - \mu_3^2) / 2\mu_2(RE_2 - 2aL_2) \dots \dots \dots (4.8)$$

For the parent particle to be thrown from infinity  $E_1 > 1$ , and Equation (4.8) reduces to the inequality

$$\mu_2^2 [RE_2^2 - 4aE_2L_2 - (r-2)L_2^2] + r\Delta(1 - \mu_3^2) - 2\mu_2(RE_2 - 2aL_2) > 0 \dots \dots \dots (4.9)$$

The above inequality can be analyzed in  $\mu_2 - \mu_3$  plane. The equality sign gives the boundary of the region for the permissible values of  $\mu_2$  and  $\mu_3$ . For the numerical values that we consider, this boundary is a hyperbola given by

$$\mu_2 = (RE_2 - 2aL_2) + [(RE_2 - 2aL_2)^2 - r\Delta(1 - \mu_3^2) \{RE_2^2 - 4aE_2L_2 - (r-2)L_2^2\}]^{1/2} / RE_2 - 4aE_2L_2 - (r-2)L_2^2 \dots \dots \dots (4.10)$$

The relevant branch of which will be decided by the following considerations.

Squaring Equation (4.1), and using  $p_2 \cdot p_3 < 0$  (future-pointing time like vectors) we see the following equation:

$$\mu_2^2 + \mu_3^2 < 1 \dots \dots \dots (4.11)$$

This is a region inside a unit circle in the  $\mu_2 - \mu_3$  plane. The inequality (4.9) requires  $\mu_2$  to be greater than the larger root or less than the smaller root given in Equation (4.10). It is the smaller root (i.e. with the negative sign for the radical) that gives the non-void intersection with the unit circle

(4.11). However,  $\mu_2$  and  $\mu_3$  should be greater than zero. The above prescription ensures that particle 1 from infinity reaches the desired splitting point, and particle 2 has negative energy. By Equation (4.2), particle 3 has greater energy than the incident particle. It now remains to ensure that particle 3 escapes to infinity. This further restricts the allowed region for  $\mu_2$  and  $\mu_3$ . For particle 3 to escape to infinity two conditions must be satisfied. The particle must bounce outside the horizon and then it should continue its motion uninterrupted. That is,

$$(i) \quad E_3 < V_3 \quad r_0 > r > r_+$$

$$(ii) \quad E_3 > V_3 \quad r > r_0$$

Where  $r_0$  is the point of split. Numerical computations to this effect show that for  $0 < \mu_3 < 1$ , and for small values of  $\mu_2$ , the particle does not escape, while for  $\mu_2$  close to the hyperbolic boundary the particle always escapes. Therefore, for a critical value of  $\mu_2$ , say  $\mu_2c'$  we have the particle scarping to infinity for  $\mu_2 > \mu_2c$ . So the allowed region. Schematic diagrams for  $\mu_2$  (max) and  $\mu_2$  (crit) are drawn. Here the numbers involved are too inconvenient to permit a figure to scale. The shaded region lying between  $\mu_2$  (max) and  $\mu_2$  (crit) is the allowed region.

now shrinks between  $\mu_2c$  and the hyperbola.

### 5- EFFICIENCY OF THE PROCESS

The most important question in the black hole energetic is the efficiency of the energy extraction process. It is therefore very pertinent to examine how efficient the Penrose process is. The maximum efficiency of the process in extracting rotational energy of the black hole () turns out to be approximately 20.7 per cent. We shall rederive this result **Chandrasekhar 1983** independently following the detailed analysis done by Parthasarathy *et al.* (1985) and show that the presence of charge on the black hole reduces the efficiency of the process. However, it further turns out that there exists no upper limit on the efficiency when one considers the process with electromagnetic

interaction. Our numerical results show that there do occur events with more than 100 per cent efficiency.

The maximum efficiency is obtained if we take the radial components of the velocities to be zero, the point of split being as close as possible to the horizon (MTW). We first derive the expression for efficiency at some  $r > r_+$  and then take the limit as  $r \rightarrow r_+$ .

$U_i$  ( $i=1, 2, 3$ ) denote the 4-velocity of the  $i$ th particle at the point of split,

$$U_1 = f_1(1, 0, 0, \Omega_1) \dots \dots \dots (5.1)$$

Where

$$F_1 = -(g_{ii} + g_{i\phi} \Omega_1)^{-1}, \dots \dots \dots (5.2)$$

$$\Omega_1 = -g_{i\phi}(1 + g_{ii}) + (-\psi(1 + g_{ii}))^{1/2} / (g_{i\phi}^2 + g_{\phi\phi}) \dots \dots \dots (5.3)$$

$\Omega_1$  is the angular velocity of particle 1 with respect to the asymptotic Lorentz frame, and we have taken  $E_1 = 1$ ,  $f_1$  is obtained by considering unit length of the 4-velocity vector  $U_1$ . At the point of split, the light cone imposes restrictions on the angular velocity  $\Omega$  of a future moving time like particle that  $\Omega_- < \Omega < \Omega_+$  where

$$\Omega_{\pm} = (-g_{i\phi} \pm \sqrt{-\psi}) / g_{\phi\phi} \dots \dots \dots (5.4)$$

The best result will be obtained by choosing the angular velocity of the second particle to be  $\Omega_2 \rightarrow \Omega_-$  and that of the third to be  $\Omega_3 \rightarrow \Omega_+$ . In the limit,

$$\mu_2 U_2 = K_2(1, 0, 0, \Omega_-), \dots \dots \dots (5.5)$$

$$\mu_3 U_3 = k_3(1, 0, 0, \Omega_+), \dots \dots \dots (5.6)$$

The conservation of 4-momentum can be rewritten as

$$U_1 = \mu_2 U_2 + \mu_3 U_3 \dots \dots \dots (5.7)$$

By algebraically manipulating the above equations we obtain

$$\mu_3 E_3 = (\Omega_1 - \Omega_- / \Omega_+ - \Omega_-)(g_{ii} + g_{i\phi} \Omega_+ / g_{ii} + g_{i\phi} \Omega_1) \dots \dots \dots (5.8)$$

The efficiency  $\eta$  is defined as

$H =$  gain in energy / input energy

$$H = \mu_3 E_3 - E_1 / E_1 = \mu_3 E_3 - 1 \quad \text{for}$$

$$E_1 = 1 \dots \dots \dots (5.9)$$

Now we take the limit as split point tends to  $r_+$ . Then

$$\mu_3 E_3 = [1 + g_{ii}]^{1/2} / 2 \dots \dots \dots (5.10)$$

For the extreme Kerr-Newman black hole ( $a^2 + Q^2 = 1$ ), the relevant  $g_{ij}$  at the horizon are given as:

$$G_{ii} = +(1 - Q^2),$$

$$g_{i\phi} = -(2 - Q^2)(1 - Q^2)^{1/2},$$

$$g_{\phi\phi} = (2 - Q^2)^2 \dots \dots \dots (5.11)$$

Putting in these values in Equation (5.10) we obtain

$$\mu_3 E_3 = [1 + (2 - Q^2)^{1/2}] / 2 \dots \dots \dots (5.12)$$

This will imply

$$\eta = [(2 - Q^2)^{1/2} - 1] / 2 \dots \dots \dots (5.13)$$

$$\text{For } Q = 0, H = \sqrt{2} - 1/2 = 0.207$$

Which is in agreement with the known result? Thus the presence of charge on the black hole decreases the maximum efficiency of the Penrose process in the absence of electromagnetic interaction (participating particles being uncharged).

### 5.1- EFFICIENCY IN THE PRESENCE OF ELECTROMAGNETIC INTERACTIONS

When we consider the participating particles being charged, the  $t$ -component of the conservation equation (5.7) will read as

$$E_1 + e_1 A_i = \mu_2 (E_2 + e_2 A_i) + \mu_3 (E_3 + e_3 A_i) \dots (5.14)$$

Here, the charges on particles can be chosen arbitrarily large and hence this will not give any upper limit on the efficiency (Parthasarathy *et al.*, 1985). In fact the term  $eAt = -eQ/r$  can assume arbitrarily large values for large  $e$ . This is borne out by the numerical example considered below.

Let us assume  $a = 0.8$ ,  $Q = 0.5$ . The particle 1 comes from infinity, and has Parameters  $\mu_1 = 1$ ,  $E_1 = 1$ ,  $l_1 = 0$ ,  $e_1 = 0$ . The split is taken to occur at  $r = 4.0$ . For  $l_2 = -10$  and  $e_2 = -50$  we give the maximum efficiency for various values of  $\mu_3$ . For  $\eta$  (max),  $\mu_2 = \mu_2$  (max) given by the hyperbolic boundary, and  $\mu_2 c$  defines the lower boundary of the permissible region. The first row of the table gives an instance when efficiency is 104 per cent.

### 6- CONCLUSION

The presence of electromagnetic fields around a black hole (inherent in the metric as in the present case or externally superposed) influences the behavior of negative energy states for charged particles

in the following two ways (**Dhurandhar & Dadhich 1984a, b**).

(a) The NES region is enlarged beyond the ergo sphere  $r = 2M$ .

(b)  $E$  can have larger negative values.

The maximum efficiency of the Penrose process for various values of  $\mu_3$ , when electromagnetic interactions are included. Conservation Equation (5.7) will read as (5.14). Both these factors contribute positively to the energy extraction process. The former brings in NES at comfortable  $r$ -values, thereby increasing the probability of larger number of events yielding energy extraction, while the latter tends to increase the energy gain per event resulting in greater efficiency. For the Kerr-Newman black hole, large negative charge on the test particle (i.e. large  $\lambda < 0$ ) causes (a), while both  $\lambda$  and  $l$  large and negative give rise.

It has been shown that the extraction of energy from the Kerr-Newman black hole is more efficient—in fact, there exists no upper bound on the efficiency when charged particles participate in the process (Table 2 shows an event of over 100 per cent efficiency)—in contrast to when uncharged particles are involved. In the latter case, the charge on the black hole reduces the maximum efficiency which is 20.7 per cent for the Kerr black hole. The electromagnetic extraction of black hole's energy is highly efficient.

As massive bodies cannot have significant charge on them, in our efficiency calculations we have taken  $Q/M = 1$ . We have hence neglected it in the metric but have retained its interaction with the test particle in the equations of motion. If a black hole acquires slight charge, our results would apply and will be indicative of the general behavior of NES and energy extraction process.

## 7- REFERENCES

[1] Abramowicz, M. A., Calvani, M., Nobili, L. 1983, *Nature*, 302, 597.  
[2] Bardeen, J. M., Press, W. H., Teukolsky, S. A. 1972, *Astrophys. J.*, 178, 347.

[3] Blandford, R. D., Znajek, R. L. 1977, *Mon. Not. R. astr. Soc.*, 179, 433.  
[4] Carter, B. 1968, *Phys. Rev.*, 174, 1559.  
[5] Chandrasekhar, S. 1983, *77ie Mathematical Theory of Black Holes*, Oxford University Press, New York.  
[6] Christodoulou, D. 1970, *Phys. Rev. Lett.*, 25, 1596.  
[7] Dadhich, N. 1983, *Phys. Lett.*, 98A, 103.  
[8] Dadhich, N. 1985, in *A Random Walk in Relativity and Cosmology*, Eds N. Dadhich, J. Krishna  
[9] Rao, C. V. Vishveshwara & J. V. Narlikar, Wiley, Eastern, p. 72.  
[10] Denardo, G., Ruffini, R. 1973, *Phys. Lett.*, 45B, 259.  
[11] Dhurandhar, S. V., Dadhich, N. 1984a, *Phys. Rev.*, D29, 2712.  
[12] Dhurandhar, S. V., Dadhich, N. 1984b, *Phys. Rev.*, D30, 1625.  
[13] Fishbone, L. G. 1973, *Astrophys. J.*, 185, 43.  
[14] Kozfowski, M., Jaroszynski, M., Abramowicz, M, A. 1978, *Astr. Astrophys.*, 63, 209.  
[15] Mashhoon, B. 1973, *Astrophys. J.*, 181, L65.  
[16] Misner, C. W., Thorne, K. S., Wheeler, J. A. 1973, *Gravitation*, W. H. Freeman, San Francisco.  
[17] Parthasarathy, S., Wagh, S. M., Dhurandhar, S. V., Dadhich, N. 1985, *Astrophys. J.*, (submitted).  
[18] Penrose, R. 1969, *Rev. Nuovo Cimento*, 1 (Special Number), 252.  
[19] Prasanna, A. R. 1983, *Astr. Astrophys.*, 126, 111.  
[20] M. Bhat, S. Dhurandhar & N. Dadhich Pringle J E. 1981, *A. Rev. Astr. Astrophys.*, 19, 137.  
[21] Reest M. I, Begelman, M. C, Blandford, R. D, Phinney, E. S. 1982, *Nature*, 295, 17.  
[22] Shakura, N. I., Sunyaev, R. A. 1973, *Astr. Astrophys.*, 24, 337.  
[23] Vishveshwara, C. 1968, *J. Math. Phys.*, 9, 1319.  
[24] Wagh, S. M., Dhurandhar, S. V., Dadhich, N. 1985, *Astrophys. J.*, 290, 12.

[25] Wald, R.. M. 1974, *Astrophys. J.*,  
191, 231.

[26] Wheeler, J. A. 1971, in *Nuclei of  
Galaxies*, Ed. D. J. K. O'Connell,  
NorthHolland, Amsterdam,p. 539.

# IJIRG