ISSN: 2455-1848
INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH & GROWTH
VOLUME-II ISSUE-VIII JUNE-2016
WWW.IJIRG.COM
IMPACT FACTOR- 2.58

Single Species Finite Population Growth With Specified Ratio In Two Age Groups

Manoj Kumar Shrivastava
Department of applied sciences, M.I.T.M. Gwalior M.P. India
E-mail- manojshrivastava2016@rediffmail.com

***Abstract**:*
In this research paper we have informed and studied models of single species finite population growth with special references of ratio in two age groups. We have taken migration rate with prescribed ratio $\alpha P_{n-1}^{(1)}$ of one age group (pre-fertile group) and $\beta P_{n-1}^{(2)}$ of second age group (fertile age group).Here α and β are strictly less than 1. The groups drown on the basis of new models are in agreement with the previous work results.
***Keywords:** population growth, single species, migration rate*

1- INTRODUCTION

In our work plan we have adopted to bring investigation and improvement of existing discrete population models ofV.K. Chaturvedi [2012] and construction of new models we have modified and extended our work from the previous work carried out by V.K. Chaturvedi. [2012]. in this chapter age based population is divided in two parts i.e. pre-fertile & fertile age groups. Population change over the time due to birth, death and dispersal of individual between separate populations The Change in first age group takes place due to births, deaths, transition and fix ratio of migration in each group of population and some numerical examples are worked out. The methodology used in this chapter in difference equations and solutions.

2- RMULATION OF THE PROBLEM

Age based population P_n may be divided in to two parts i.e.

$P_n = P_n{}^{(1)} + P_n{}^{(2)}$............. (1)

Where

$P_n^{(1)}$= population of infants juveniles (pre-fertile)

$P_n^{(2)}$= population of the fertile age group

The change in population of first age group occurs due to births deaths, migration and transition from one age group to the second age group.

Change the population $P_n^{(1)}$is given by

$$\Delta P_n^{(1)} = P_n^1 - P_{n-1}^1$$

Or

$$\Delta P_n^{(1)} = BP_{n-1}^{(2)} - D_n P_{n-1}^{(1)} - T_n P_{n-1}^{(1)} + \alpha P_{n-1}^{(1)}$$
...... (2)

Where

B = Birth rate (Uniform Throughout)

$D_n^{(1)}$= Death rate in group one in n^th^ generation

T_n = Transition rate from $P_{n-1}^{(1)}$to $P_{n-1}^{(2)}$ in $n^{th}$the generation

$\alpha P_{n-1}^{(1)}$ = Migration rate in one group n^{th} generation.

[Sinceα is the prescribed ratio of population of the one age group of n^{th} generation and α is strictly less than 1]

Hence

$$P_n^{(1)} - P_{n-1}^{(1)} = BP_{n-1}^{(2)} - D_n P_{n-1}^{(1)} - T_n P_{n-1}^{(1)} + \alpha P_{n-1}^{(1)}$$

$P_n^{(1)}$= (1-D_n-T_n +α) $P_{n-1}^{(1)}$+ B

Now let

$$(1- D_n- T_n+\alpha) = X_n$$

Then

$P_n^{(1)}$= $X_n P_{n-1}^{(1)}$ +B $P_{n-1}^{(2)}$ ....(3)

Similarly

$$\Delta P_n^{(2)} = P_n^{(2)} - P_n^{(2)}$$

i.e.

$$\Delta P_n^{(2)} = T_n P_{n-1}^{(1)} - D_n^{(2)} P_{n-1}^{(2)} + \beta P_{n-1}^{(2)}$$

Where

$D_n^{(2)}$ = Death rate in group two in n^{th} generation.

T_n = Transition rate from $P_{n-1}^{(1)}$ to $P_{n-1}^{(2)}$ in n^{th} generation

β $P_{n-1}^{(2)}$ = Migration rate in second group in n^{th} generation.

[Since β is the prescribed ratio and population of the in second age group of n^{th} age group and β is strictly less than 1]

$P_n^{(2)} - P_{n-1}^{(2)}$= $TP_{n-1}^{(2)}$– $D_n^{(2)}$ $P_{n-1}^{(2)}$+β $P_{n-1}^{(2)} P_n^{(2)}$= (1– $D_n^{(2)}$+β) $P_{n-1}^{(2)}$+ $T_n P_{n-1}^{(1)}$ ---------- (4)

Let

$$P_{n-1}^{(1)} = \mu P_{n-1}^{(2)}$$

[Since μ is prescribed ratio and, 0< μ < 1]

Then

$P_n^{(2)}$= (1-$D_n^{(2)}$ +β) $P_{n-1}^{(2)}$ + μT_n $P_{n-1}^{(2)}$

$$P_n^{(2)} = (1- D_n^{(2)}+\mu T_n+ \beta)\, P_{n-1}^{(2)}$$

$P_n^{(2)}$ =$Y_n P_{n-1}^{(2)}$.................................. (5)

Where

Y_n= (1-$D_n^{(2)}$ +μ T_n+β)

3- SOLUTION OF DIFFERENCE EQUATION

Solution of difference equation (5) can be given as V.P. Saxena (2011)

$$P_n^{(2)} = \prod_{i=1}^{n} Y_i P_0^{(2)}$$

Replacing n by n-1, and we get

$$P_{n-1}^{(2)} = \prod_{i=1}^{n-1} Y_i P_0^{(2)}$$

Putting these values in eq. (3) we get

$$P_n^{(1)} = X_n P_{n-1}^{(1)} + B\left[\prod_{i=1}^{n-1} Y_i P_0^{(2)}\right]$$

.... (6)

The solution of the difference eq. (3) we get

$$P_n^{(1)} = \prod_{i=1}^{n} X_i P_0^{(1)} + B\sum_{r=1}^{n}\prod_{i=r+1}^{n} X_i \prod_{i=1}^{n-1} Y_i P_0^{(2)}$$

.... (7)

Special Case – 1

Let $Y_1 = Y_2 = Y_3 \ldots\ldots\ldots\ldots Y_n = Y$

Then,

$$P_n^{(2)} = Y^n P_0^{(2)}$$

(a) Step-1, if n= 1

$$P_1^{(2)} = YP_0^{(2)}$$

(b) Step-2 , if n= 2

$$P_2^{(2)} = Y^2 P_0^{(2)}$$

(c) Step-3, if n= 3

$$P_3^{(2)} = Y^3 P_0^{(2)}$$

(d) Step-4, if n= 4

$$P_4^{(2)} = Y^4 P_0^{(2)}$$

(e) Step-5, if n= 5

$$P_5^{(2)} = Y^5 P_0^{(2)}$$

3.1- NUMERICAL EXAMPLES

Numerical Calculations have been carried out using Mat. Lab 6.5 Programming and also for groups

Example-1

(a) Let $P_0^{(1)} = 1500, P_0^{(2)} = 2500$,M $= 20$

(b) Let $P_0^{(1)} = 1000, P_0^{(2)} = 2000$,M $= 30$

Then

$$P_1^{(2)} = 2500Y$$

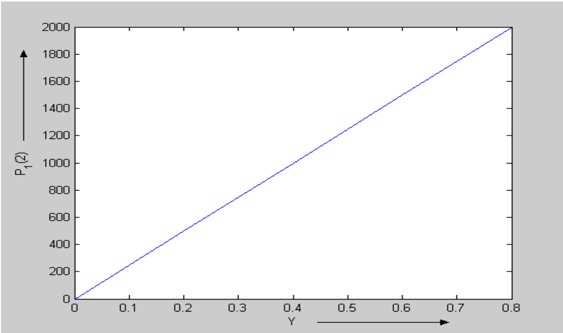

Figure-1 graph between $P_1^{(2)}$ and Y

Special Case – 2

Let $X_1 = X_2 \dots\dots\dots\dots\dots X_n = X$

$B_1 = B_2 \dots\dots\dots\dots\dots . B_n = B$

Then

$$P_n^{(1)} = X^n P_0^{(1)} + BP_0^{(2)}$$

(a) **Step-1, if** n=1

$$P_1^{(1)} = XP_0^{(1)} + BP_0^{(2)}$$

(b) **Step-2,** if n=2

$$P_n^{(1)} = X^2 P_0^{(1)} + B(X+Y)P_0^{(2)}$$

(c) **Step-3,** if n=3

$$P_3^{(1)} = X^3 P_0^{(1)} + B(X^2 + XY + Y^2)P_0^{(2)}$$

(d) **Step-4,** if n=4

$$P_4^{(1)} = X^4 P_0^{(1)} + B(X^3 + X^2Y + XY^2 + Y^3)P_0^{(2)}$$

(e) **Step-5** if n=5

$$P_5^{(1)} = X^5 P_0^{(1)} + B(X^4 + X^3Y + X^2Y^2 + XY^3 + Y^4)P_0^{(2)}$$

3.2- NUMERICAL EXAMPLE

Numerical Calculations have been carried out using Mat. Lab 6.5 (2007) Programming and also for group

Example-1

(a) $P_0^{(1)}$ =1500 , B=0.15, $P_0^{(2)}$ = 2500

(b) $P_0^{(1)}$ =1000 , B=0.25, $P_0^{(2)}$ = 2000

Then

$$P_1^{(1)} = 1500X + 375$$

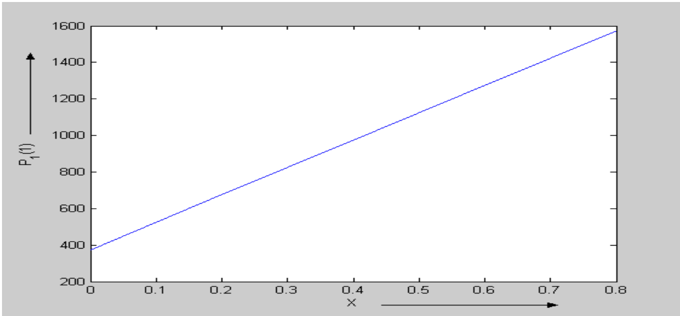

Figure-2 graph between $P_1^{(1)}$ and Y

$$P_1^{(1)} = 1000X + 500$$

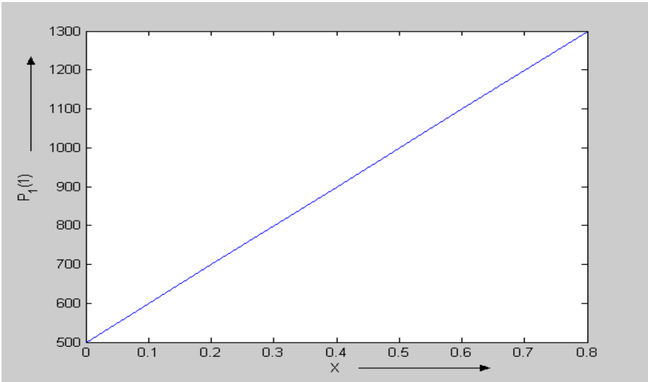

Figure-3graph between $P_1^{(1)}$ and Y

Example-2

(a) $P_0^{(1)}$ =1500 , B=0.15, $P_0^{(2)}$ = 2500

(b) $P_0^{(1)}$ =1000 , B=0.25, $P_0^{(2)}$ = 2000

Then

$$P_2^{(1)} = 1500X^2 + 375(X + Y)$$

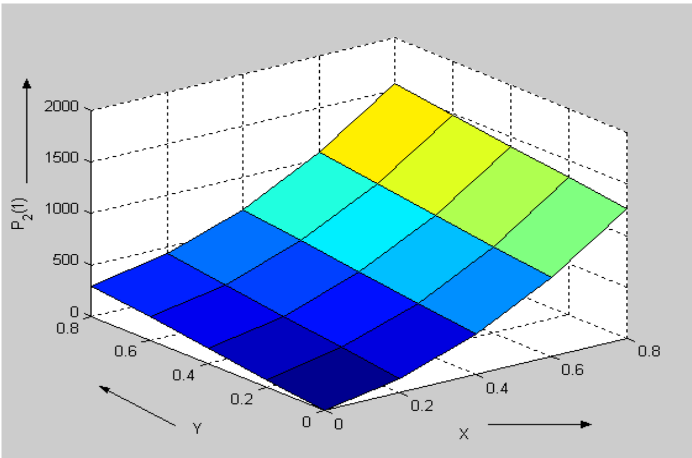

Figure-4 graph between $P_2^{(1)}$ and Y

$$P_2^{(1)} = 1000X^2 + 500(X+Y)$$

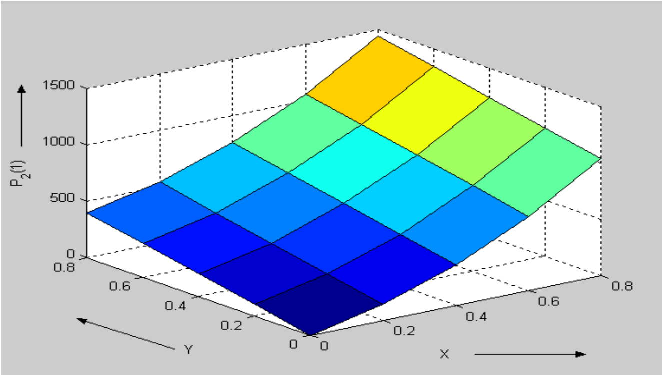

Figure-5graph between $P_2^{(1)}$ and Y

4- CONCLUSION

This paper presents new modified models of previous models Dr. V.K. Chaturvedi (2012) with births, deaths, transition and Migration is prescribed ratio of o two age group of single species. It helps to protect the population and species in wild life. The modified models. Seems to contribute to estimate the single species finite population growth in any region and the requirement of resources. The graphs between population and the parameters x or y provides the pattern of growth two population graphs separately. Two dimensional graphs useful for wild life management.

5- REFERENCES

[1] -Renshaw, Eric (1991): Modeling Biological , University Press. pp. 6–

[2]-V.P. Saxena (2011) application of special functions is Modeling animal population of finite size. Natt. Acad.Sci. Lett. Vol.34, No. 9&10.

[3]- G.D. Vaishya, (2007): Saxena's 1-function and it Biological applications (Ph.D.) thesis Jiwaji University:144-184

[4]- V.K. chaturvedi (2012): mathematical study of single and Two interchanging species with special preference to Protected wild life.

[5]- Kapur J. N. (1979): "Optimal harvesting of animal Populations"Indian Journal of pure and applied Mathematics, Vol.10 (7) pp.890-909.