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PATTERN AND GROWTH OF ANIMAL POPULATION WITH THREE AGE GROUPS

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Abstract
In this research paper we have introduced one species and divided it in to three age groups. In the first age group we have taken migration zero and in second and third groups we have taken prescribed ratio of migration rate we conclude that there is no change the growth in the finite age groups.

Keyword: *-prescribed ratio, Population and parameters.*

Introduction
In this research paper two type of species are taken one type of species considered two age group and other type of species considered three age group (Post reproductive) is of the same order on the remaining two these types of models are divided by Vaishya (2007) and Chaturvedi V.K (2012) In this paper we extended the same we carried out more investigations with migration of first age group is zero and fixed ratio of migration of second and third age group and some important numerical examples are worked out.The methodology used in this paper in difference equations.

Formulation of Problem
Medium and large animal species with longer generation of life, the population has to be divided in at least three age group we can taken

$An \quad = An^{(1)} + An^{(2)} + An^{(3)}$-------------- (1)

$An^{(1)}$= Population of infants and juveniles (pre-fertile)

$An^{(2)}$= Adult population in the fertile age group

$An^{(3)}$= Population of aged non-fertile

The changein population of first age group will take due to births deaths and migration is zero, therefore change in the population $An^{(1)}$ is given by the equation

$\Delta An^{(1)} = An^{(1)} - A_{n-1}^{(1)}$--------------------- (2)

The governing equation can be written as

$\Delta An^{(1)} = B_1 A_{n-1}^{(2)} - D_1 A_{n-1}^{(1)} - T_1 A_{n-1}^{(1} - q_1 Q A_{n-1}^{(1)} + m_1$.. (3)
Where

B_1 = Birth rate

D_1 = Death rate

T_1 = Transition rate from $A_{n-1}^{(1)}$to $A_{n-1}^{(2)}$

Q = Predator population.

q_1 = Predation rate in first group.

According from difference equation (1)&(3)

Or$An^{(1)} - A_{n-1}^{(1)} = B_1 A_{n-1}^{(2)} - D_1 A_{n-1}^{(1)} - T_1 A_{n-1}^{(1)} - q_1 Q A_{n-1}^{(1)} + 0$

[Where migration of first age group is zero]

Or$An^{(1)} = (1 - D_1 - T_1 - q_1 Q) A_{n-1}^{(1)} + B_1 A_{n-1}^{(2)}$-----------(4)

Similarly, wehave

$\Delta An^{(2)} = An^{(2)} - A_{n-1}^{(2)}$ ---------- (5)

The governing difference eq. is

$\Delta An^{(2)} = -D_2 A_{n-1}^{(2)} - T_2 A_{n-1}^{(2)} - q_2 Q A_{n-1}^{(2)} + m_2 + T_1 A_{n-1}^{(1)}$....................(6)

We are taking migration $m_2 = \alpha A_{n-1}^{(2)}$

[Whereα is prescribed ratio of Population of second age group and strictly less than 1 i.e. $(0 < \alpha < 1)$]

According to the difference equation (5) & (6)

Or$A_n^2 - A_{n-1}^{(2)} = -D_2 A_{n-1}^{(2)} - T_2 A_{n-1}^{(2)} - q_2 Q A_{n-1}^{(2)} + \alpha A_{n-1}^{(2)} + T_1 A_{n-1}^{(1)}$

Or $An^{(2)} = (1 - D_2 - T_2 - q_2 Q + \alpha) A_{n-1}^{(2)} + T_1 A_{n-1}^{(1)}$----------(7)

Where

D_2 = Death rate

T_2 =Transition rate from $A_{n-1}^{(2)}$ to $A_{n-1}^{(3)}$

Q = Predator population

q_2 = Preduation rate in second age group.

α = Prescribed ratio of Population of second age group and strictly less than 1 i.e. $[0 < \alpha < 1]$

Similarly we can write

$An^{(3)} = (1 - D_3 - q_3 Q - T_3) A_{n-1}^{(3)} + T_2 A_{n-1}^{(3)} + m_3$

$An^{(3)} = (1 - D_3 - q_3 Q - T_3) A_{n-1}^{(3)} + T_2 A_{n-1}^{(2)} + \beta A_{n-1}^{(3)}$

$An^{(3)} = (1 - D_3 - T_3 q_3 Q + \beta) A_{n-1}^{(3)} + T_2 A_{n-1}^{(2)}$--------(8)

[We are taking migration $m_3 = \beta A_{n-1}^{(3)}$]

(Where$\beta$are prescribed ratio of population of third age group and strictly less than 1 i.e.$[0 < \beta < 1]$

Where

D_3 = Death rate

q_3 = Predation in third age group.

Q = Predator population

T_3 = Transition rate for age group of third.

Solution of the problem

From eq. (4)

Let$(1 - D_1 - T_1 - q_1 Q) = X$

Then eq. (4) will become

$An^{(1)} = XA_{n-1}^{(1)} + B_1 A_{n-1}^{(2)}$---------------(9)

Again from eq. (7)

Let$(1 - D_2 - T_2 - q_2 Q + \alpha) = Y$

Then eq. (7) will become

$An^{(2)} = YA_{n-1}^{(2)} + T_1 A_{n-1}^{(1)}$------------------------(10)

From Eq. (8)

Let$(1 - D_3 - T_3 q_3 Q + \beta) = Z$

Then eq. (8) will become

$An^{(3)} = ZA_{n-1(}^{3)} + T_2 A_{n-1}^{(2)}$------------------(11)

Now we find the solution of eq. (9), (10) &(11) by induction method.

$A_1^{(1)} = XA_0^{(1)} + B_1 A_0^{(2)}$ -------------------- (12)

$A_1^{(2)} = \mathrm{y} A_0^{(2)} + T_1 A_0^{(1)}$ ------------------------ (13)

$A_1^{(3)} = ZA_0^{(3)} + T_2 A_0^{(2)}$ ------------------------- (14)

$A_2^{(1)} = XA_1^{(1)} + B_1 A_1^{(2)}$

$$= X\left\{XA_0^{(1)} + B_1 A_0^{(2)}\right\} + B_1\left\{yY + T_1 A_0^{(1)}\right\}$$

Or$A_2^{(1)} = (X^2 + B_1 T)\ A_0^{(1)} + B_1 (X + Y)\ A_0^{(2)}$------------ (15)

$A_2^{(2)} = yYA_1^{(2)} + T_1 A_1^{(1)}$

Putting the value of $A_1^{(2)}$&$A_1^{(1)}$ from eq. (12) & (13)

$A_2^{(2)} = Y\ (YA_0^{(2)} + T_1 A_0^{(1)} + (XA_0^{(1)} + B_1 A_0^{(2)})\ T_1$

$= (Y^2 + B_1 T_1)\ A_0^{(2)} + T_1 (X+Y)\ A_0^{(1)}$---------------------(16)

Again $A_3^{(1)} = X\ A_2^{(1)} + B_1\ A_2^{(2)}$

Putting the value of $A_2^{(1)}$&$A_2^{(2)}$ from (15) & (16)

$A3^{(1)} = X\ [(X^2+B_1T_1)\ A_0^{(1)} + B_1\ (X+Y)\ A0^{(2)}] + B_1\ [(Y^2+B_1T_1)\ A0^{(2)} + T_1\ (X+Y)\ A_0^{(1)}]$

$0r\ \ A_3^{(1)} = X^3 A_0^{(1)} + B_1 T_1\ (2X+Y)\ A_0^{(1)} + B_1 A_0^{(2)} \sum_{r=0}^{2} X^{2-r} Y^r + B_1^{(2)}\ T1\ A_0^{(2)}$......... (17)

Similarly We Find

$A_3^{(2)} = Y\ A_2^{(2)} + T_1 A_2^{(1)}$

Putting the value of $A_2^{(1)}$ and $A_2^{(2)}$ from (13) & (14)

Or $A_3^{(2)} = Y^3 A_0^{(2)} + B_1 T_1 (2Y+X) A_0^{(2)} + T_1 A_0^{(1)} \sum_{r=1}^{3} X^{3-r}.Y^{r1} + B_1 T_1^2 A_0^{(1)}$ ... (16)

Again , $A4^{(1)} = XA_3^{(1)} + B_1 A_3^{(2)}$

Now substituting the value $A_3^{(1)}$ & $A_3^{(2)}$ in gave eq.

$$A_4^{(1)} = X^4 A_0^{(1)} + B_1 T_1 A_0^{(1)} \sum_{r=0}^{2} (3-r)\, X^{2-r}.Y^r + B_1^2 T_1 A_0^{(1)} + B_1 A_0^{(2)} \sum_{r=1}^{3} X^{4-r}.Y^{r-1} + B_1^2 T_1 (2X+2Y) A_0^{(2)} \text{.......} (18)$$

Again, $A_4^{(2)} = YA_3^{(2)} + T_1 A_3^{(1)}$

Now Substitution the value of $A_3^{(2)}$ & $A_3^{(1)}$ in above eq.

$$A_4^{(2)} = Y^4 A_0^{(2)} + B_1 T_1 A_0^{(2)} \sum_{r=1}^{3} (4\text{-}r)\, X^{r-1}.Y3^{-r} + B_1^2 T_1^2 A_0^{(2)} + T_1 A_0^{(1)} \sum_{r=1}^{4} X^{4-r}.Y^{r-1} + 2B_1 T_1^2 (X+Y) A_0^{(1)} \text{.......} (19)$$

Again, $A5^{(1)} = XA_4^{(1)} + B_1 A_4^{(2)}$

Now Substitute the value of A_4 (1) & A_4 (2) from eq. (18) & (19)

$$A_5^{(1)} = X^5 A_0^{(1)} + B_1 T_1 A_0^{(1)} \sum_{r=1}^{4} (5-r)\, X^{4-r} Y^{r-1} + B_1^2 T_1^2 (3X+2Y) A_0^{(1)} + B_1 A_0^{(2)} \sum_{r=1}^{5} X^{5-r} Y^{r-1} + B_1^2 t_1 (3X^2+4XY+3Y^2) A_0^{(2)} + B_1^3 T_1^2 A_0^{(2)} \text{.......} (20)$$

From, $A_5^{(2)} = YA_4^{(2)} + T_1 A_4^{(1)}$

Substitute the value of $A_4^{(1)}$ & A_4 (2) in above equation.

$$A_5^{(2)} = Y^5 A_0^{(2)} + B_1 T_1 A_0^{(2)} \sum_{r=0}^{3} (4-r)\, Y^{3-r} X^r + B_1^2 T_1^2 (2X+3y) A_0^{(2)} + T_1 A_0^{(1)} \sum_{r=1}^{5} X^{r-1} Y^{5-r} + B_1 T_1^2 (3X^2 + 4XY3Y^2) A_0^{(1)} + B_1^2 T_1^3 A_0^{(1)} \text{.......} (21)$$

Numerical example

Numerical calculation done with the help of mat lab 7.5 (2007) programming and also all the graphs drawn with same.

Case -I. Example -1 $q_1 = 0.001$, $q_2 = 0$, $s_1 = \frac{20}{200}$, $D2 = \frac{15}{100}$

$T_1 = T_2 = T = \frac{1}{100}$, $B_1 = \frac{25}{100}$, A_0 (1) = 1000, A_0 (2) = 2000.

Q = 300, 400, 500,

(a) $A_1 (1) = x A_0 (1) + B_1 A_0 (2)$

$A_1^{(1)} = 1000x + 500$ (b) $A_2 (1) = x (x A_0 (1) + B_1 A_0 (2)) + B_1 [yA_0 (2) + T_1 A_0 (1)]$

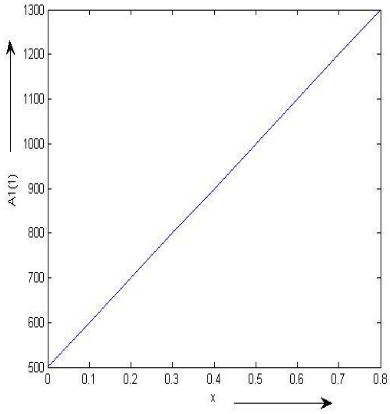
$= (x_2 + B_1 T) A_0 (1) + B_1 (x+y) A_0 (2)$

$A_2^{(1)} = 1000 x^2 + 500 (x+y) + 2.5$

(c) $A^3 (1) = 1000 x^3 + 5x + 2.5y + 500 xy + 500 x^2 + 500y2 + 1.25$

Example-1, Figure (a)

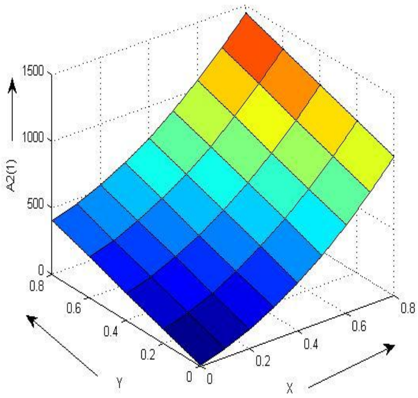
$A_1^{(1)}$= 1000x + 500



Graph between X and $A_1(1)$.

Example-1, Figure (b)

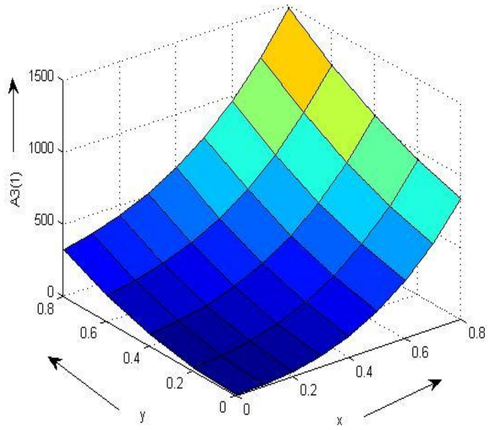
$A_2^{(1)}$= 1000 x^2 + 500 (x+y) + 2.5



Graph between X, Y and $A_2(1)$

Example -1, Figure (c)

A^3 (1) = 1000 x^3 + 5x + 2.5v + 500 xv + 500 x^2 + 500v2 + 1.25



Graph between X, Y and $A_3(1)$

Conclusion

The figures of numerical example. Give the India. Thatin three significant age groups. The population of predators does not change in compression to the main population of medium size species. The migration rate of different age groups is the main factor. The graphs between Population and parameters x and y provided the pattern of growth of three population groups separately.

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