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A NOTE ON THE MITTAG-LEFFLER TYPE FUNCTION

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Abstract
In this paper the authors derive the results based on the Mittag-Leffler type function. Some special cases of interest are also discussed.

Introduction
The importance of Mittag-Leffler functions in physics is steadily increasing. It is simply said that deviations of physical phenomena from exponential behavior could be governed by physical laws through Mittag-Leffler functions (power law). The Mittag-Leffler function.

$$E_{\alpha}(x)=\sum_{r=0}^{\infty}\frac{x^{r}}{\Gamma(\alpha r+1)},\quad(\alpha>0) \tag{1.1}$$

and its generalized form

$$E_{\alpha,\beta}(x)=\sum_{r=0}^{\infty}\frac{x^{r}}{\Gamma(\alpha r+\beta)},\quad(\alpha,\beta>0) \tag{1.2}$$

A generalization of (1.1) and (1.2) was introduced by Prabhakar in terms of the series representation

$$E_{\alpha,\beta}^{\gamma}(x)=\sum_{r=0}^{\infty}\frac{(\gamma)_{n}x^{r}}{r!\Gamma(\alpha r+\beta)}\quad,(\alpha,\beta,\gamma\in C,\mathrm{Re}(\alpha)>0) \tag{1.3}$$

Where $(\gamma)_n$ is Pochammer's symbol defined by

$$(\gamma)_{n}=\gamma(\gamma+1).......((\gamma+(n-1)),n\in N,\gamma\neq 0.$$

It is an entire function of order $\rho=[\mathrm{Re}(\alpha)]^{-1}$.
A generalization of (1.3) was defined by Sharma as

$${}_{p}M_{q}^{\alpha}(a_{1},\ldots,a_{p};b_{1},\ldots,b_{q};x)={}_{p}M_{q}^{\alpha}(x)=\sum_{r=0}^{\infty}\frac{(a_{1})_{r}\ldots(a_{p})_{r}}{(b_{1})_{r}\ldots(b_{q})_{r}}\frac{x^{r}}{\Gamma(\alpha r+1)} \tag{1.4}$$

Where $\alpha\in C,\mathrm{Re}(\alpha)>0$ and $(a_j)_r$ and $(b_j)_r$ are the Pochammer symbols. The detailed information of this series is given in.

A generalization of (1.3) was defined by Sharma as

$$ {}_pM_q^{\alpha,\beta}(a_1,\ldots,a_p;b_1,\ldots,b_q;x) = {}_pM_q^{\alpha,\beta}(x) = \sum_{r=0}^{\infty} \frac{(a_1)_r\ldots(a_p)_r}{(b_1)_r\ldots(b_q)_r} \frac{x^r}{\Gamma(\alpha r+\beta)} \tag{1.5} $$

Where $\alpha,\beta \in C, \mathrm{Re}(\alpha) > 0$ and $(a_j)_r$ and $(b_j)_r$ are the Pochammer symbols. The detailed information of this series is given in.

- Left-sided Riemann-Liouville fractional integral

$$ (I_{0+}^{\alpha} f)(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt, \mathrm{Re}(\alpha) > 0. \tag{1.6} $$

- Right-sided Riemann-Liouville fractional integral

$$ (I_{-}^{\alpha} f)(x) = \frac{1}{\Gamma(\alpha)} \int_x^{\infty} (t-x)^{\alpha-1} f(t)dt, \mathrm{Re}(\alpha) > 0. \tag{1.7} $$

Fractional Differentiation and Integration of the generalized M-Series

In this section results connecting the function defined by (1.6) and the Riemann-Liouville fractional integrals and derivatives are presented in the form of theorems given below:

Theorem 1.1- let $\alpha > 0, \beta > 0, a \in R$ and I_{0+}^{α} is the left-sided Riemann-Liouville fractional integral operator then there holds the formula:

$$ (I_{0+}^{\alpha}\{t^{\gamma-1}\,{}_rM_s^{\beta,\gamma}(a\,t^{\beta})\})(x) = x^{\alpha+\chi-1}\,{}_rM_s^{\beta,\alpha+\gamma}(a\,x^{\beta}) \tag{2.1} $$

Proof:

By using the definition of left sided Riemann-Liouville fractional integral (1.6) and definition (1.5), we arrive at the desired result.

On a similar fashion we can prove another theorem given below.

Theorem 1.2- let $\alpha, \beta, \gamma > 0, a \in R$ and I_{-}^{α} be the right-sided Riemann-Liouville fractional integral operator then there holds the formula:

$$ (I_{-}^{\alpha}\{t^{-\alpha-\gamma}\,{}_rM_s^{\beta,\gamma}(a\,t^{-\beta})\})(x) = x^{-\chi}\,{}_rM_s^{\beta,\alpha+\gamma}(a\,x^{-\beta}) \tag{2.2} $$

Proof:

By using the definition of right sided Riemann-Liouville fractional integral (1.7) and definition (1.5), we arrive at the desired result.

Remarks: If we set $r = s = 0$ in above theorems, we get the results given by Saxena and Saigo.

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