

Energy Module In The Life Of Stars: A Study

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1- BACK GROUND

The historical back ground of different to settle down to a neutron star equilibrium configuration. The contraction would continue without end beyond the event horizon and ultimately what is left is nothing but a gravitational field so intense so intense that no matter or signals of any kind are allowed to escape to infinity and bring information about the fate of the collapsing matter. The body has fallen deep inside the event horizon. Event horizon is one way membrane. Inside it, the gravitational field is so powerful that nothing can escape outside. Even light emitted at any point inside the event horizon is dragged inward irrespective of the direction of emission. However, light emitted at any point outside the event horizon can escape to infinity if it is suitably aimed at. Because of the nature of black holes they can properly detected only through the effects of their gravitational field on the closely surrounding objects provided they can be observed.

It is now an established fact that powerful radiation emission can take place after swallowing of mater by a black hole. Many astrophysicists are convinced that X-rays comprise the most significant observable emission from the velocity of a black hole. There are others who are of the view that one of the few ways in which X-rays can be produced is by mass accretion onto collapsed orbits specially black holes. Such a mass accretion can take place in various ways. The accreting mass may be falling

radically or spiraling down on isolated black hole system. So far as the observational aspect is concerned, the good way to infer about the existence of black hole is to capitalize on a double-star system where the missing companion is suspected to be a black hole and is so near to a normal star that it draws in matter from its companion. Such a flow from one star to another is well known in close binary system and a strong emission in the X-rays region is expected if one of the components is a neutron star or a black hole. For such a binary system, material flowing from one star would have too much angular momentum to be able to fall directly on to a compact companion. It would instead from a spinning disc, in which the matter spirals inward. The energy liberated by the disc could again emerge as X-rays.

2- FRAMES TO ISOLATED BLACK HOLES

Firstly, we consider the accretion of mass to an isolated black hole. It has been found out in chapter-IV that in the cases of Kerr black hole, there exists just outside the ergo sphere a circular orbit $r=(3+)m/2$ where $r=r_{ms}$, $r=r_{mb}$ and $r=r_{ph}$ coincide. Hence, it appears that at this distance, the infilling gas due to dragging of inertial frames will be swung into orbital rotation about the hole. Its angular velocity, then, can be obtained (with negative sign). This is given for maximally rotating holes,

$$\Omega = \frac{m + \sqrt{mr}}{(r + m)\sqrt{mr} + r^2 m^2}$$

and

$$\Omega]_r = \frac{3+\sqrt{5}}{2} + m \underline{\Omega} \frac{1}{5m}$$

The binding energy in the orbit is given by,

$$E_{\text{bind}} = 1 - E/\mu \underline{\Omega} 0.14$$

Due to radiation, the particle may lose its energy and then it would gradually spiral inward through the ergo sphere to settle down to the last circular orbit near the horizon. As the gas approaches the horizon, its angular velocity as seen from infinity must approach the angular velocity of the horizon for (a = m).

$$\Omega = \Omega_{\text{horizon}} 1/(2m).$$

From the following Table (1).

Table - 1

	$\rho = 1.5,$				$-5.91m < \lambda < 4.91m$			
λ	-5.9m	-3.9m	-1.9m	.1m	2.1m	2.8m	3m	4.1m
+z	3.77	3.15	2.52	1.9	1.28	1.06	.99	0.65

We come to the conclusion that as the particle spiral down from $r=(3+\sqrt{5})m/2$ to the last circular orbit $r = m$, there are marked and abrupt changes in the frequency shift of omission. As the particle falls from the orbit $r = (3+\sqrt{5})m/2$, there is slight decrease in the value of the frequency shift but it abruptly increases as $r = 2m$. We conclude that the region near $r = (3+\sqrt{5})m/2$, is a region of increased activity. The increase is more of abrupt for values of r less than $2m$. There is fantastically high spread in $(1+z)$ among the photons emitted at small value of r near $r = m$. This is consistent with the strong gravitational field in that region. At values of r greater than $(3+\sqrt{5})m/2$, the frequency shift decreases slowly. From a close examination of the table No.1, it is clear that frequency shift of emitted radiation is greater than 1 for values of impact parameters in the range of $-7m < \lambda < 2m$.

Now, we come to the case of slowly rotating isolated black hole. Accretion of a photon or a particle to a black hole may occur broadly in two ways. A particle falling into the black hole or it may occur broadly in two ways. A particle falling into the black hole or it may be that on approaching the hole, it gets deflected. The particle that goes in and gets caught gives

energy and angular momentum to the black hole. If the particle going in and gets deflected by the hole, it picks up energy and angular momentum from the whole. A particle entering the ergo sphere, if it is properly powered, can escape to infinity. A real particle in the ergo sphere must always change its position regardless of whether it eventually escapes to infinity or enters the event horizon to collapse. From the ergo sphere a particle can always send a signal out to infinity. The dynamics of the geometry of the ergo sphere can be probed in the case of Kerr metric. Penrose (49) have shown that it is possible to extract energy out of the zone. For this purpose, he has suggested the following steps:

- (1) A small object with rest plus kinetic energy E_1 is shot into this region.
- (2) It is allowed to explode (or turn) on its rocket engine) in such a way that that the disintegration product (or, equivalently, the rocket ejects) crosses the event horizon and gets accreted to the hole.
- (3) The residual mass is allowed to re-emerge from the surface of infinite red shift with total energy E_2 .
- (4) The process is so arranged that E_2 exceeds E_1 .

The energy $E_2 - E_1$ can be said to have been extracted from the rotational energy of the black hole in the sense that angular

momentum of the black hole always decreases in such a process. If the “energy gain process” is repeated again and again, the black hole will be losing its angular momentum. The rotation of the black hole will be slower and slower and $(a/m) \rightarrow 0$. In the equatorial plane, the one way membrane (event horizon) expands and coalesces with the infinite red shift surface, wiping out the ergo sphere. The Kerr metric in the limit reduces to the perturbed Schwarzschild metric. It appears from the Chapter-IV that the orbits just outside the ergo sphere in the limit settles down to orbits given by respectively outside the event horizon of the perturbed. There is also another way of obtaining this perturbed metric. For the orbits given by the angular velocity of orbital rotation and the binding energy are:

$$\Omega = a/5.72m,$$

$$E_{\text{bind}} \rightarrow 0.18$$

Thus, a particle spiraling in from $r = \infty$ towards a black at the last circular orbit at $r = 2m(1+a)$ radiates a fraction $1 - \sqrt{2/3}$ or 18 percent of the rest mass. In the case of particle spiraling in a maximally rotating Kerr black hole radiates 42 percent of the rest mass before arriving at the last circular orbit.

Now we proceed to investigation the frequency shift of radiation emitted near the orbit. We get from

$$2mr(L-aE)^2 = K^2 E^2 r^2 (r^2 + a^2),$$

And $(k^2 + 1)E^2(r^2 + a^2) = L^2 + \Delta\mu^2$,
Equation is so transformed as

$(\sqrt{2}m) (L/E) = \sqrt{2}ma + K \sqrt{r(r^2 + a^2)}$
Substituting for k, we get

$$\lambda = a + \frac{r\sqrt{r}}{\sqrt{3r-2m}}$$

Substituting $a = m\alpha$ and $r = 2m(1+\alpha)$ and neglecting terms containing α^2 and higher power of α , we get λ

$$\lambda = m [\sqrt{2} + \{ (3/\sqrt{2}) + 1 \} \alpha]$$

Again, making the substitution $a = m$

and $r = 2m(1+\alpha)$ and approximating by neglecting α^2 and higher powers of α , we get -

$$\lambda_1 = -m[4 + 3\alpha] \text{ and } \lambda_2 = m[4 + \alpha]$$

Then for outgoing photons emitted near $r=2m(1+\alpha)$ to escape, λ must satisfy the condition -

$$\lambda_1 < \lambda < \lambda_2$$

We can say that for the outgoing photons to escape, λ can have the following values, $\lambda = -m(4 + k\alpha)$, where $k < 3$,

or, $\lambda = m(4 + k_1\alpha)$, where $k_1 < 1$,
For the frequency shift for the first set of values, we get from by making approximation as in the following expression:

$$1+z = \frac{0.18(4 + 20\alpha + 3\sqrt{2}\alpha)}{4\alpha}$$

For the values of λ given above

$$1+z = \frac{0.18(4 + 20\alpha + 5\sqrt{2}\alpha)}{4\alpha}$$

For the desired value It can be easily be verified that for a frequency shift greater than 1, $\alpha < 0.23$. As α becomes smaller and smaller, it can be easily verified becomes higher and higher. For the ingoing photons the emitted radiation will escape if $\lambda > m(4+a)$ or $\lambda = m(4+k_2\alpha)$ where $k_2 > 1$. We get for frequency shift an expression which is same.

Hence, we conclude that in the black hole represented by the perturbed metric, it is possible for the particle in the last circular stable orbit $r=2m(1+\alpha)$ to emit radiation which can escape to infinity.

3- ACCRETING GAS IN THE BINARY SYSTEM: KEPLERION ORBITS

In the binary system, gas can flow from the atmosphere of the ordinary star into its companion hole. Also a super massive hole ($10^7 M_{\odot} \lesssim M \lesssim 10^{11} M_{\odot}$) at the centre of a galaxy, due to its high mass and the large gas density accretes much more than a hole of ordinary mass in a normal interstellar region. The accreting gas in a binary system and in the center of a galaxy has very high specific angular momentum. Hence, the accretion is far from spherical. Instead of falling inward radically or roughly radically, the gas elements go into Keplerian orbits around the hole, forming gas disc analogous to Saturn's rings. However, the density in accreting disc is far greater than the density in Saurian ring. The presence of viscosity in accreting disc removes the angular momentum permitting the gas to spiral gradually into the hole. Viscosity also heats the gas which causes it to radiate and this radiation is largely the X-rays in binary system and ultraviolet and blue light in super massive holes. The angular momentum removed by the viscosity is transported by viscous stresses from the inner part of the disc to the outer part and then carried away by passing gas. The total energy radiated by a unit mass of gas during its passage inward through the disc is approximately equal to the gravitational binding energy of the unit mass when it reaches the inner edge of the disc. For a black hole, the inner edge of the accreting disc is at the last stable circular orbit.

We have seen that in the case of maximally rotating Kerr black hole, there exists a stable circular orbit $r_{ms} = (3 + \sqrt{5})m/2$, just outside the ergo sphere. Hence, it is just possible that the accreting gas first forms a disc with its inner edge at the circular orbit $r = (3 + \sqrt{5})m/2$, where its binding energy is given by,

$$E_{bind} \simeq 0.14$$

Here the presence of viscosity in accreting disc will remove a part of the angular momentum permitting the gas to spiral through the ergo sphere, the angular

momentum of the gas spiraling through the ergo sphere is still high enough.

Hence the gas elements go into Keplerian orbit to form a disc with its inner edge at the last circular orbit. The binding energy there is given by,

$$E_{bind} \simeq 0.42$$

Here also the presence of viscosity will remove the remaining angular momentum and the gas finally falls in the hole.

Now we consider the black hole represented by the perturbed metric. In this case also the accreting disc will be formed with inner edge at the last circular orbit $r_{ms} = 2m(1+\alpha)$. The binding energy is given by:

$$E_{bind} \simeq [\sqrt{2/3}][1 + (\alpha/8\sqrt{2})]$$

The viscosity will remove the angular momentum permitting the gas to spiral gradually will cause it to radiate. The orbital period in the last circular orbit in this case is given by:

$$P_{min} = \frac{4\pi}{\Omega} = \frac{4\Omega \times 5.27m}{\alpha} \approx \frac{23\pi m}{\alpha}$$

Thus, the orbital period in this case is greater than the period in case of maximally rotating Kerr hole and also in the case of non-rotating hole. P_{min} in these cases are given as follows:

$$P_{min} = 12\pi\sqrt{6}m, \text{ for non-rotating hole.}$$

$$P_{min} = 4\pi m, \text{ for maximally-rotating hole.}$$

Swanyayev has given a test for rotation of the black hole as follows:

“A black hole is non-rotating if $P_{min} = 12\pi\sqrt{6}m$ or rotating if $4\pi m \lesssim P_{min} \lesssim 12\pi\sqrt{6}m$. It appears that test particle falls in case of slowly rotating black hole represented by the perturbed metric represented.

Lastly, it appears that the circular orbits near the event horizon of the perturbed metric can explain the formation and nature of rings round the Saturn and other planets in our galaxy.

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